EINSTEIN RING:
COSMIC HORSESHOE

TE FOTOGRAFEREN MET EEN KLEINE TELESCOOP?

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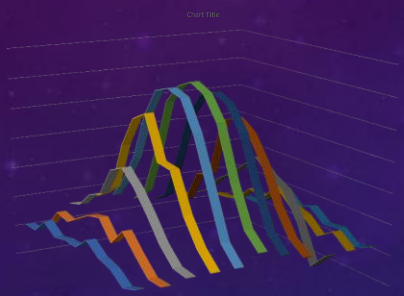
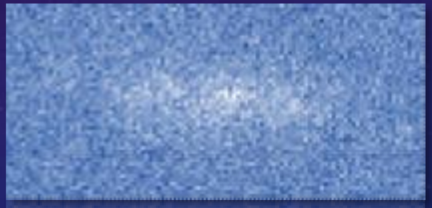
Leidse Weer- en Sterrenkundige Kring
Oude Sterrewacht, Leiden, 2024-09-17


Hubble Space Telescope opname van de *Cosmic Horseshoe*.

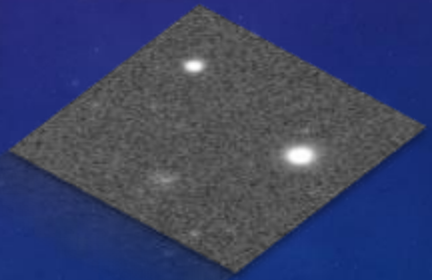
Introductie



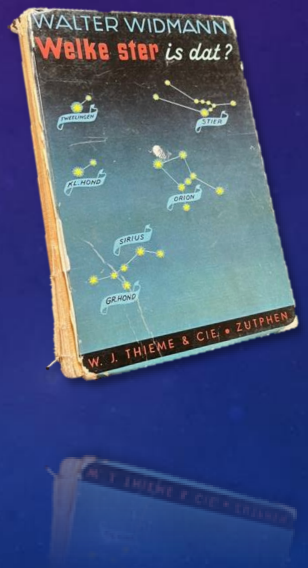
Techniek



Signaal / Ruis



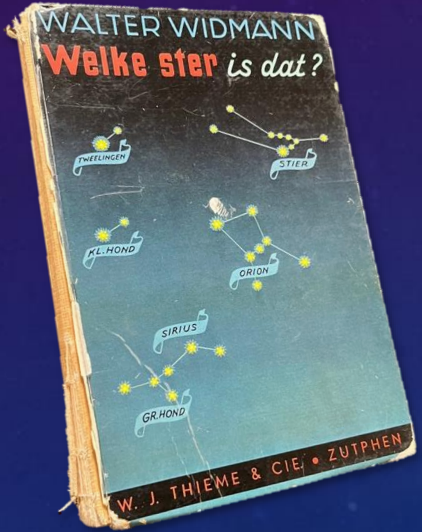
Gravitatie en de Ring



INTRODUCTIE

1970-1980

30 mm lens
Ringen van Saturnus!



2010

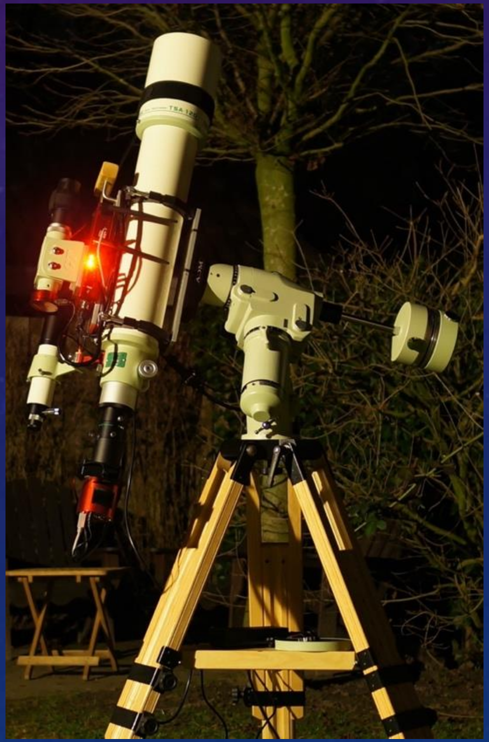
90 mm William Optics
Azimuthaal, visueel

2014-01-28

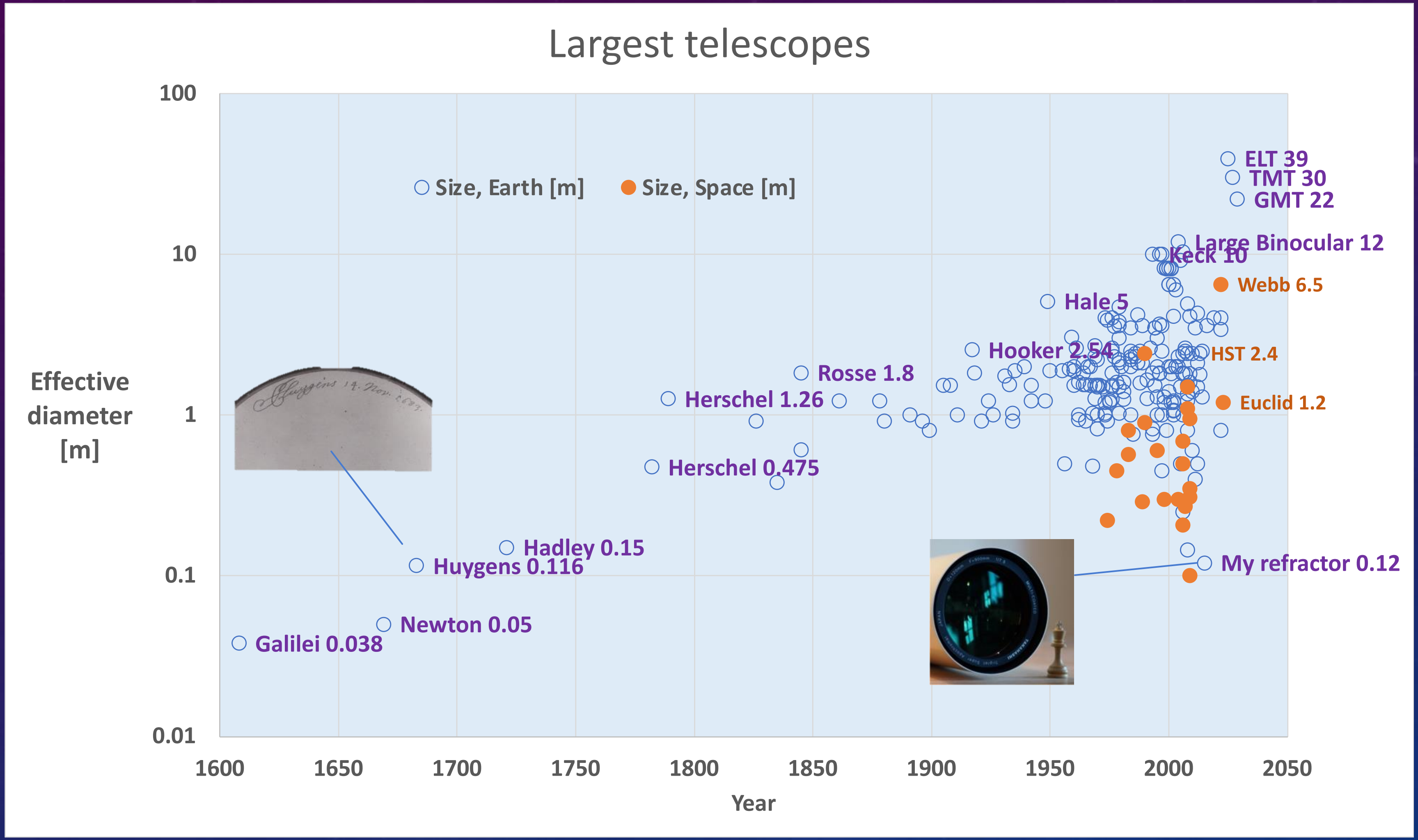
M82 met supernova
2.5 s belicht, niet gevolgd – jammer!

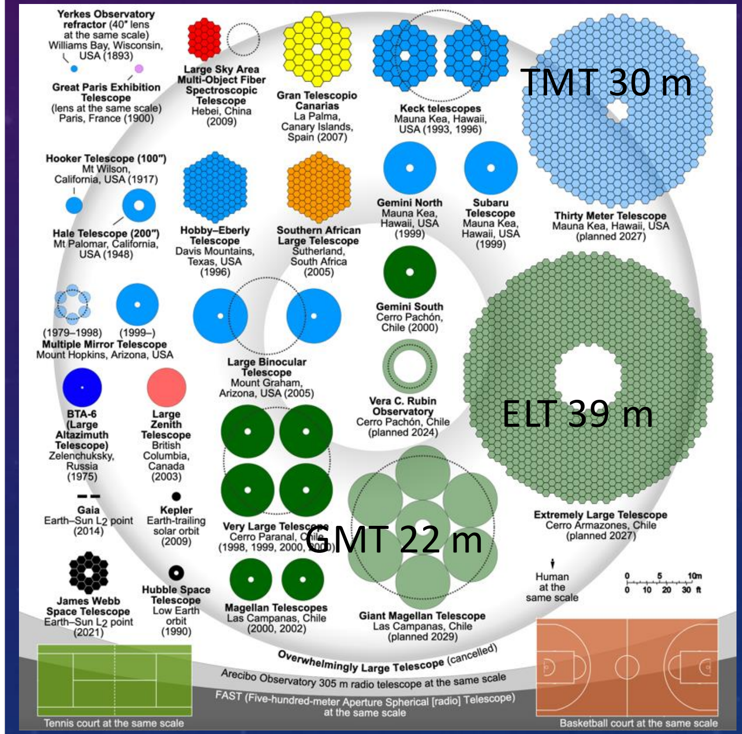
2015-

120 mm Takahashi
Parallactisch, fotografisch



Grootste telescopen in de historie



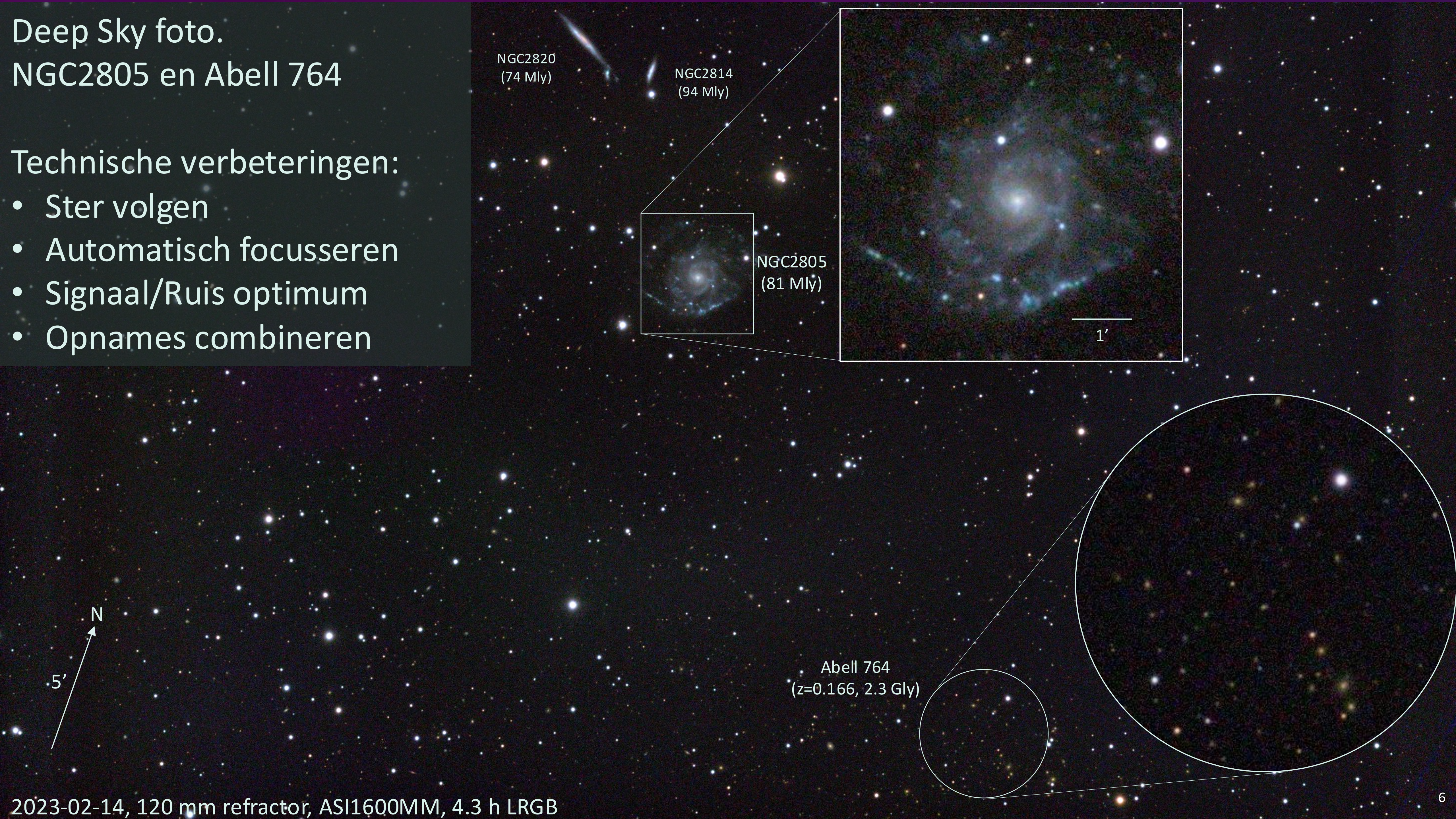


Source: Wikipedia

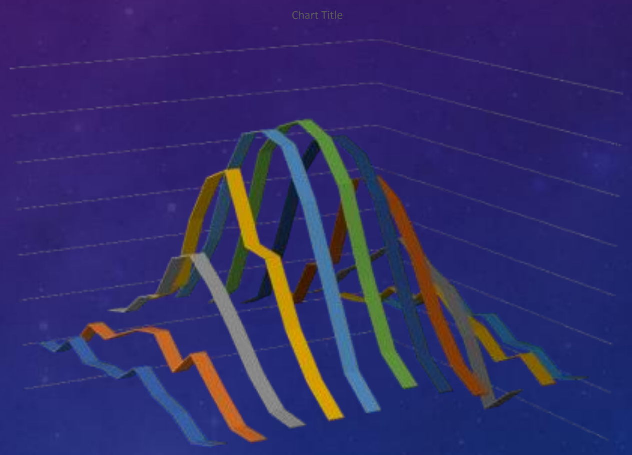
Deep Sky foto.
NGC2805 en Abell 764

Technische verbeteringen:

- Ster volgen
- Automatisch focusseren
- Signaal/Ruis optimum
- Opnames combineren

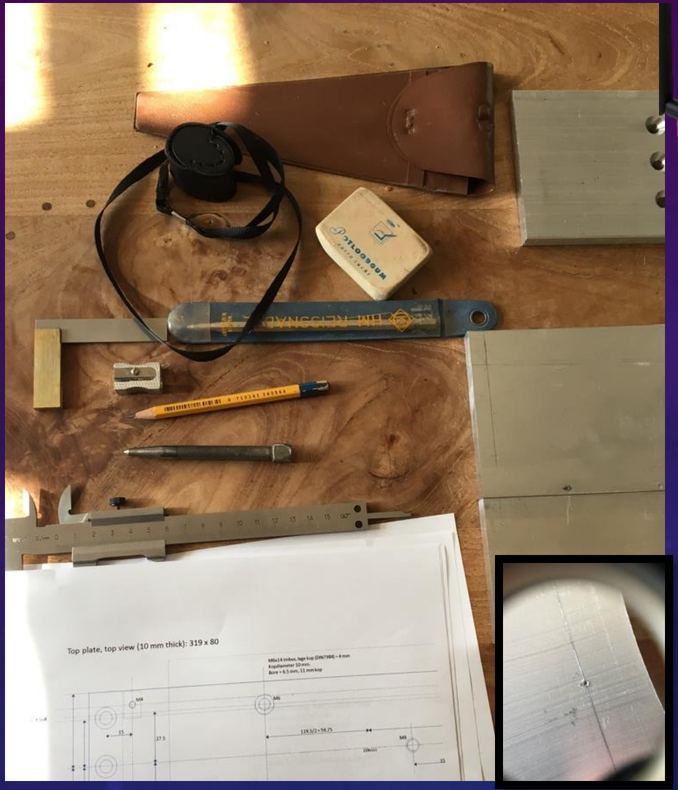


2023-02-14, 120 mm refractor, ASI1600MM, 4.3 h LRGB



TECHNIEK

VERBETERINGEN

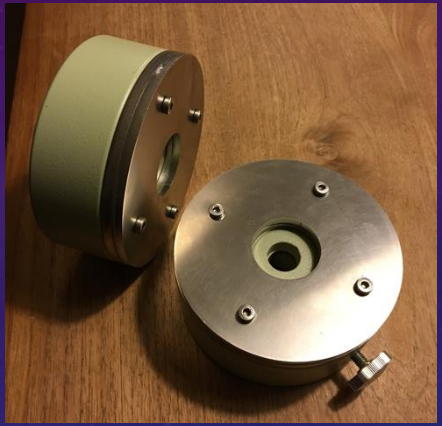


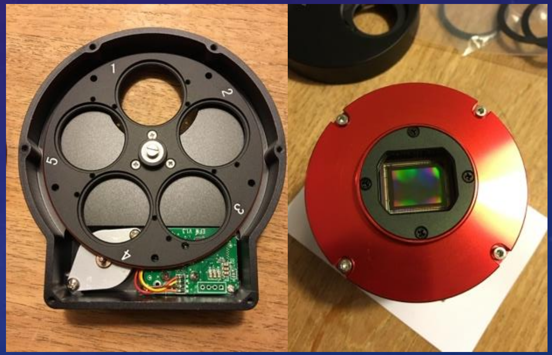
Compacte constructie met berekend zwaartepunt

Computer en vocht-temperatuursensors

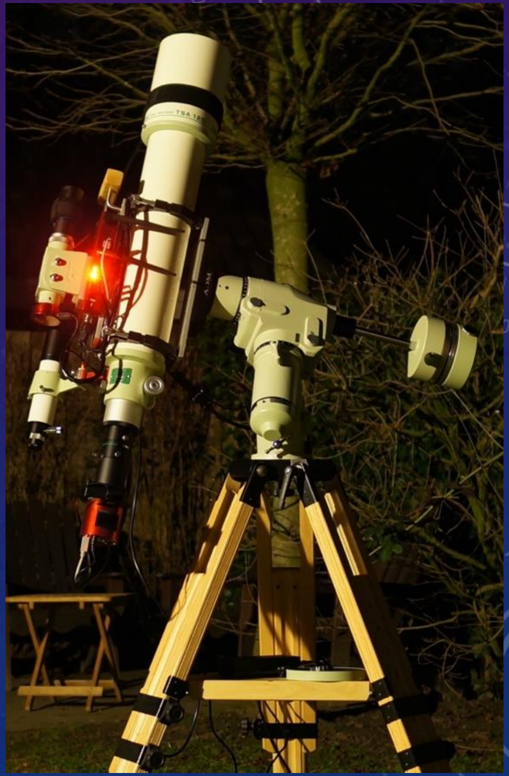
Rigide telescoopbeugels

Extra contragewicht



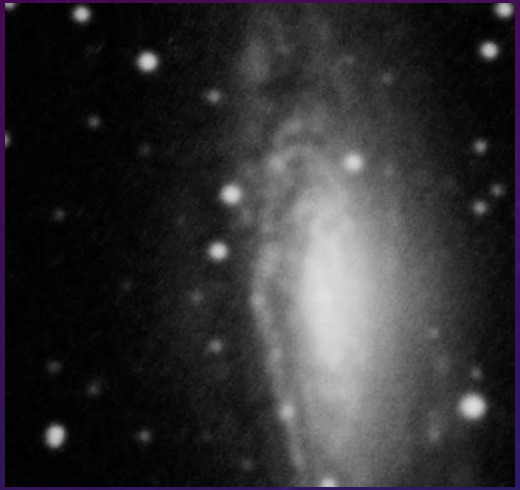


Gekoelde CMOS camera met LRGBHα filters



Betrouwbare veldopstelling autonoom via WiFi

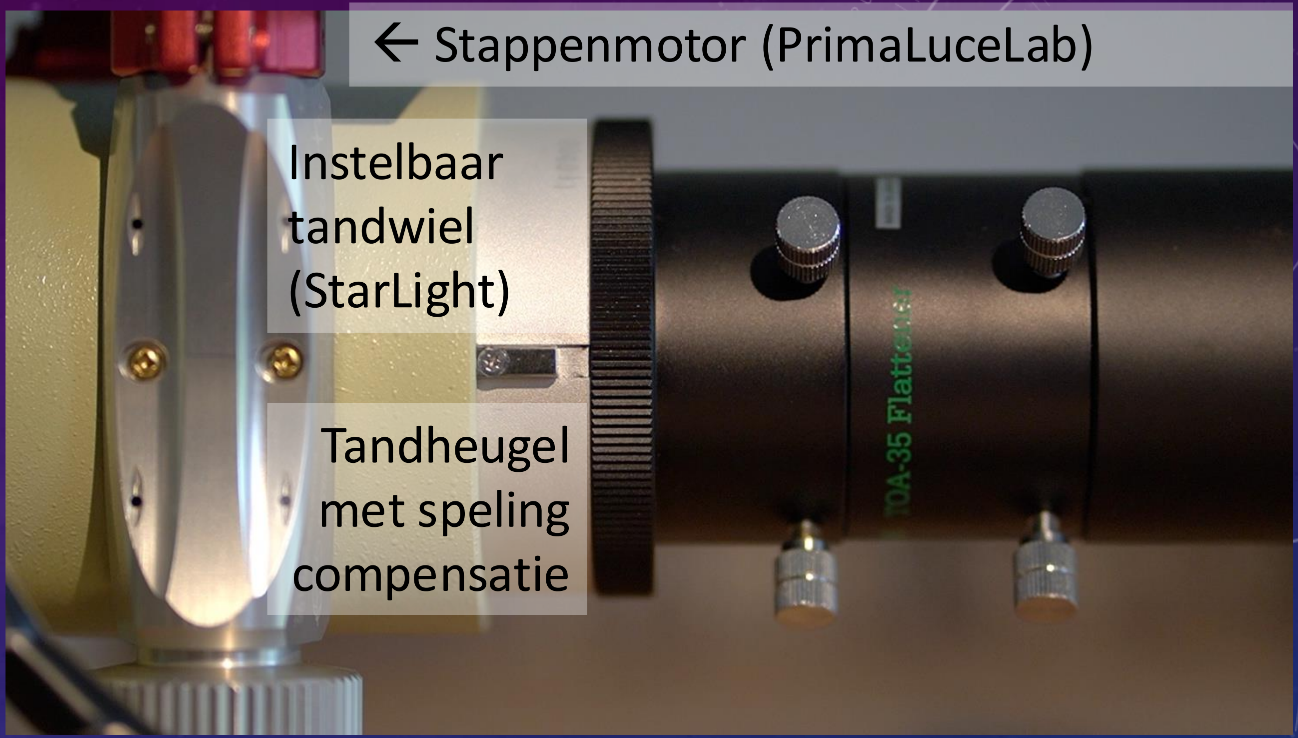
AUTOMATISCHE FOCUS



Onvoldoende

Automatisch en herhaald

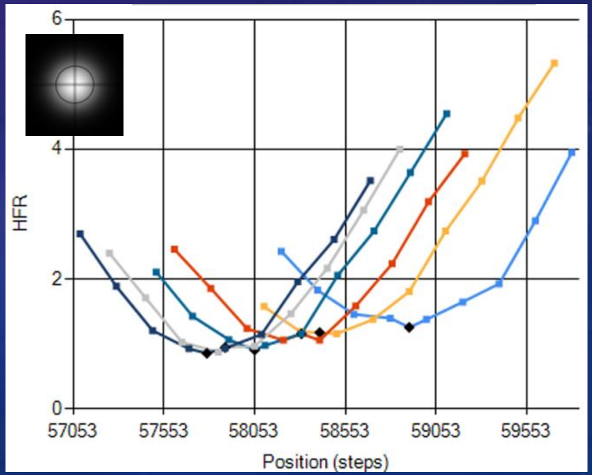
- De Half-Flux-Radius (HFR) bepaald beste focus instelling in Sequence Generator Pro (SGP) →
- Focusseer-positie is temperatuur-bepaald.

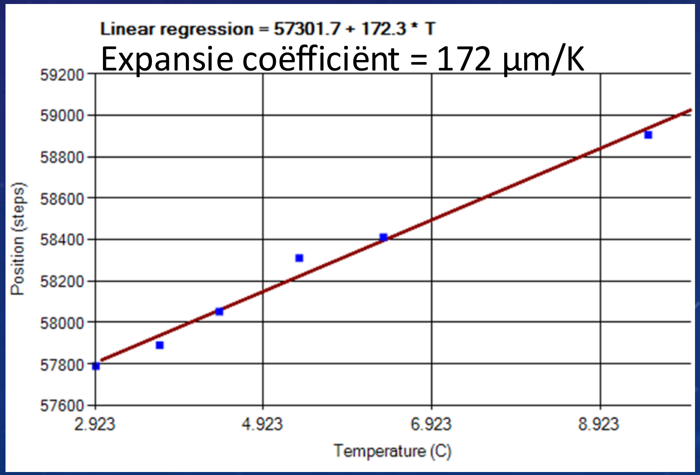


← Stappenmotor (PrimaLuceLab)

Instelbaar tandwiel (StarLight)

Tandheugel met speling compensatie





Linear regression = 57301.7 + 172.3 * T

Expansie coëfficiënt = 172 µm/K

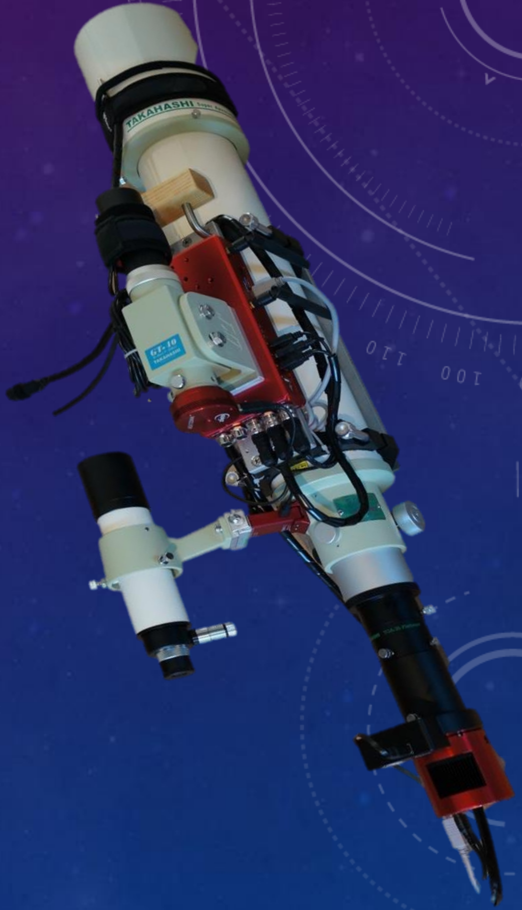
110 JAAR VERBETERINGEN

Lichtgevoelige chemie

CMOS sensor

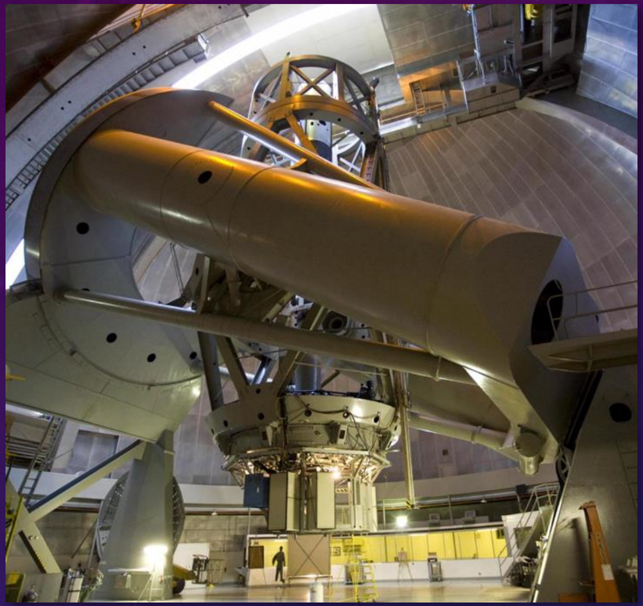
1.5 m
Mt. Wilson
telescoop
in 1908

120 mm
telescoop



1952: 5 m Hale Telescoop, Mount Palomar, USA.

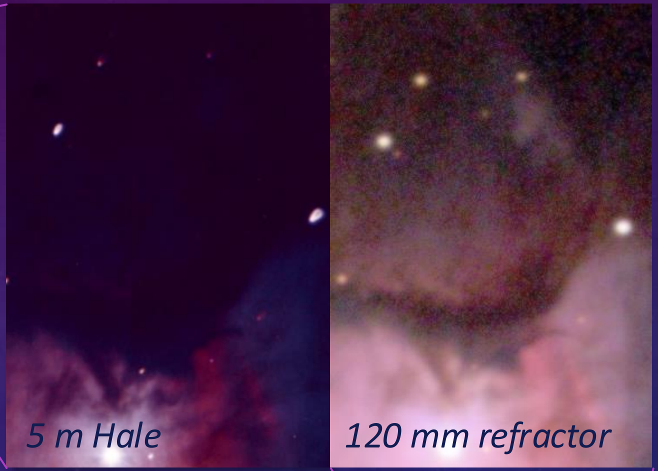
M42 Orion Nebula



5 m, f/3.3 Hale telescoop op Mount Palomar, USA.

Orion nevel (M42) foto met 5 m Hale Telescoop.

Coma aberratie!



2019-02-25 M42 in Orion met Takahashi 120 mm

3L2R2G2B x 120 s (18’) en ASI1600MM camera op -10 ˚C vanuit Berg en Dal.

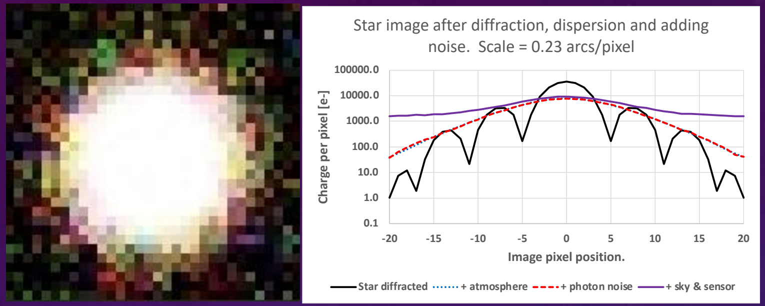
Grotere telescopen niet zomaar scherper!

Dit komt door onze atmosfeer.

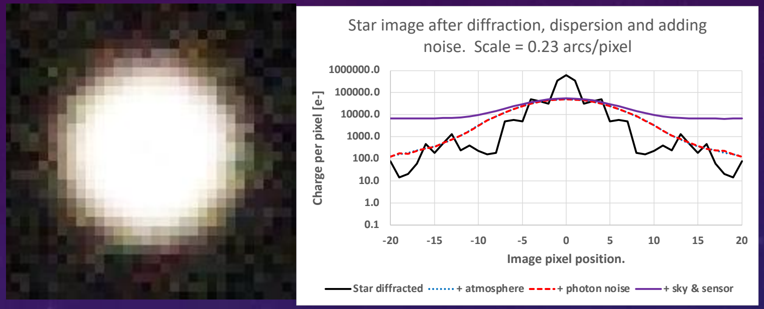
Twee oplossingen:
- in de ruimte (HST, Webb, Euclid, etc.)
- met adaptieve optiek (alle grote telescopen op Aarde)

Gesimuleerd met een witte ster (F0V spectrum) van magnitude 14 door convolutie van atmosferische dispersie en optische diffractie op een 0.23 arcs pitch sensor. Een centrale obstructie in de telescoop om diffractie effecten te versterken. 240 s belicht en achtergrond magnitude 19 / arcs^2.

2” seeing

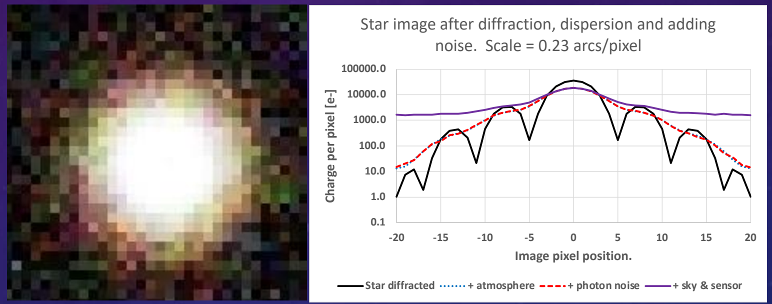


120 mm

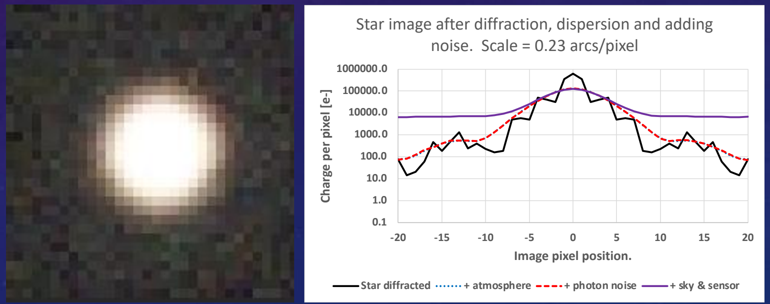


250 mm

1” seeing

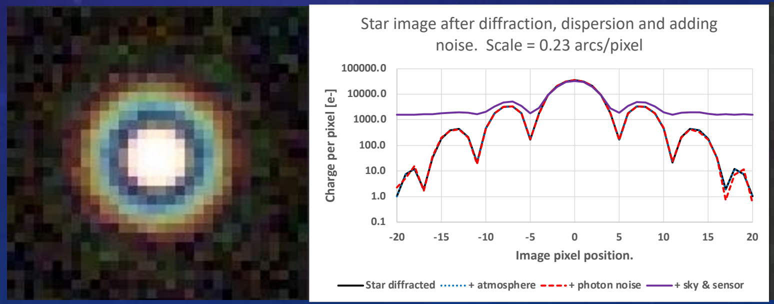


120 mm

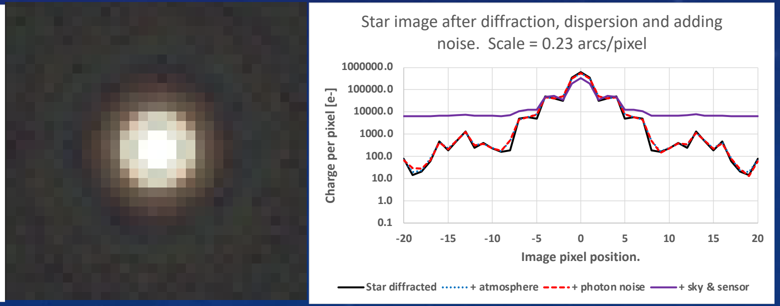


250 mm

0” seeing (geen atmosfeer)



120 mm



250 mm

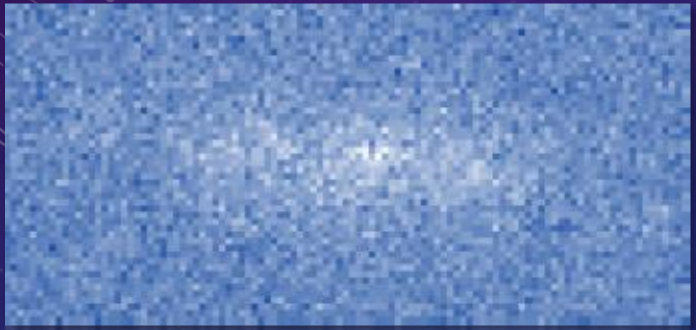
2022: 120 mm refractor versus Webb

Stephan’s Quintet

2018-2020 2.6 h met
120 mm refractor

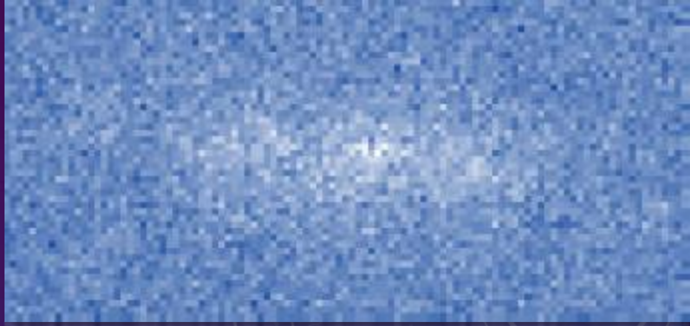
2022
6.5 m Webb

2018-2020 2.6 h met 0.12 m refractor (beeld: 69’ x 52’)



SIGNAAL / RUIS

SIGNAAL / RUIS



Beste signaal-ruis verhouding nodig om Cosmic Horseshoe te kunnen zien.

Diverse ruisbronnen

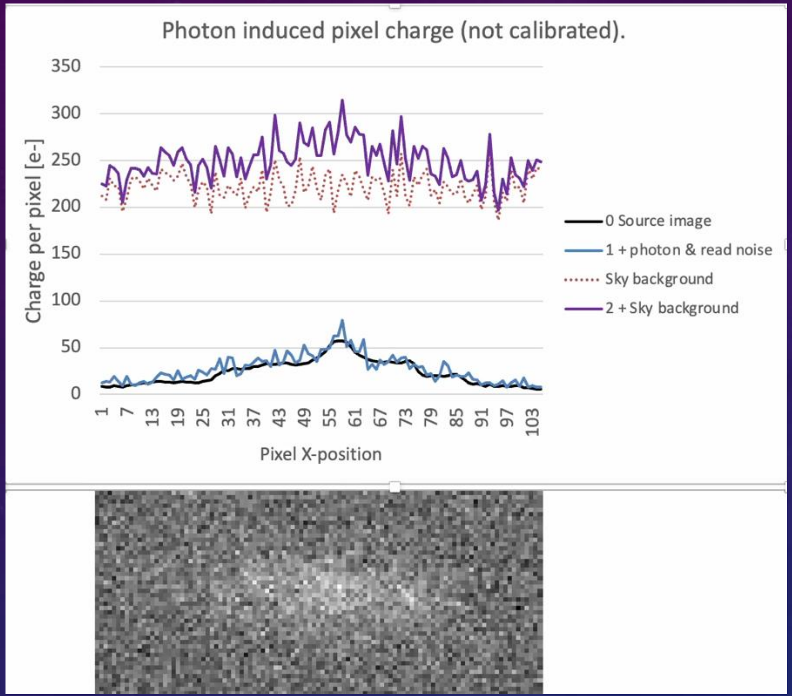
- Uitleesruis camera
- Thermische ruis camera
- Poisson- of signaalruis

- Atmosferische achtergrondruis

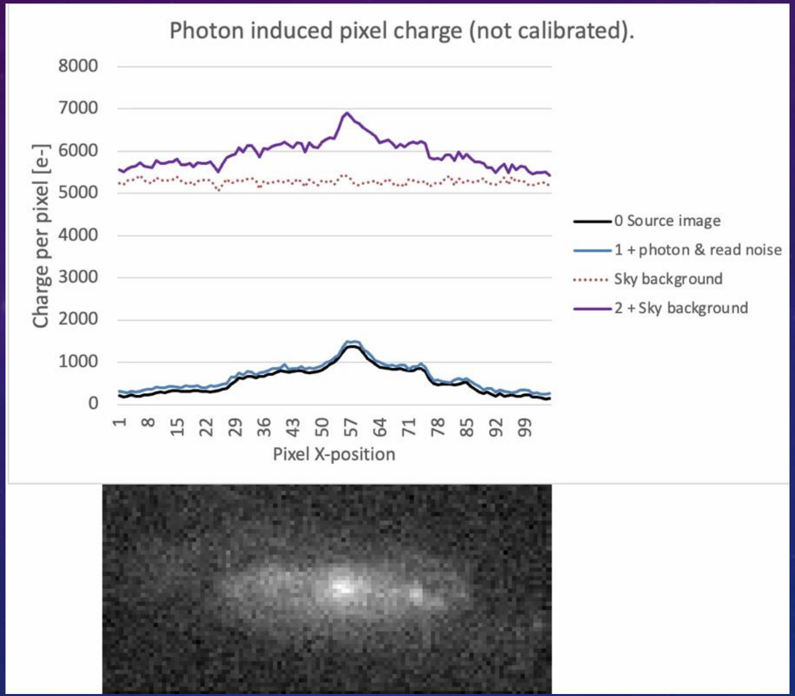
Beter Signaal/Ruis door:

- Optimalisatie van belichtingstijd per subframe
- Koelen camera
- Grotere telescoop
- Langer belichten
- Meer Flats en Darks
- Donkere locatie

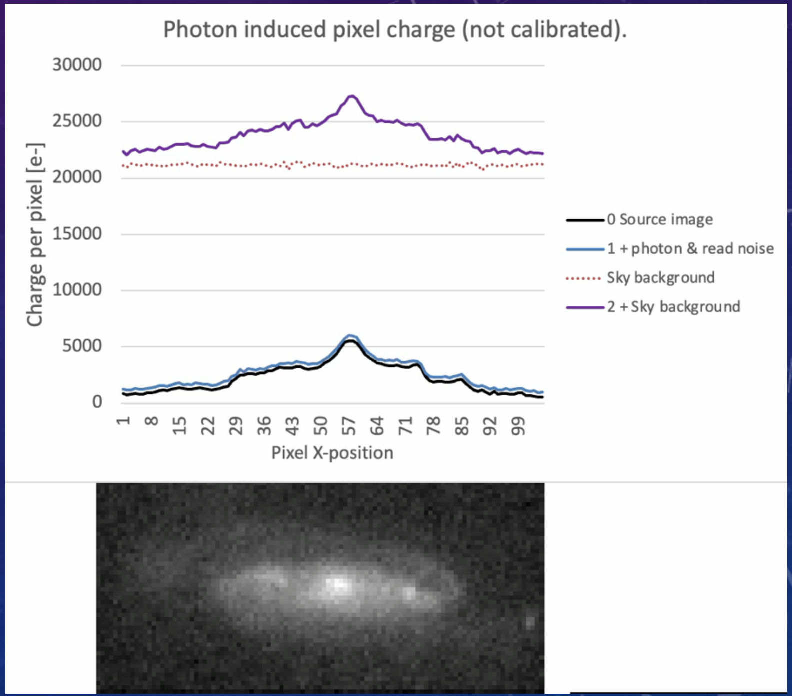
LANGER BELICHTEN → HOGER SIGNAAL/RUIS



240 s (x = 1)



1.6 h (x = 24) → 5x betere S/N



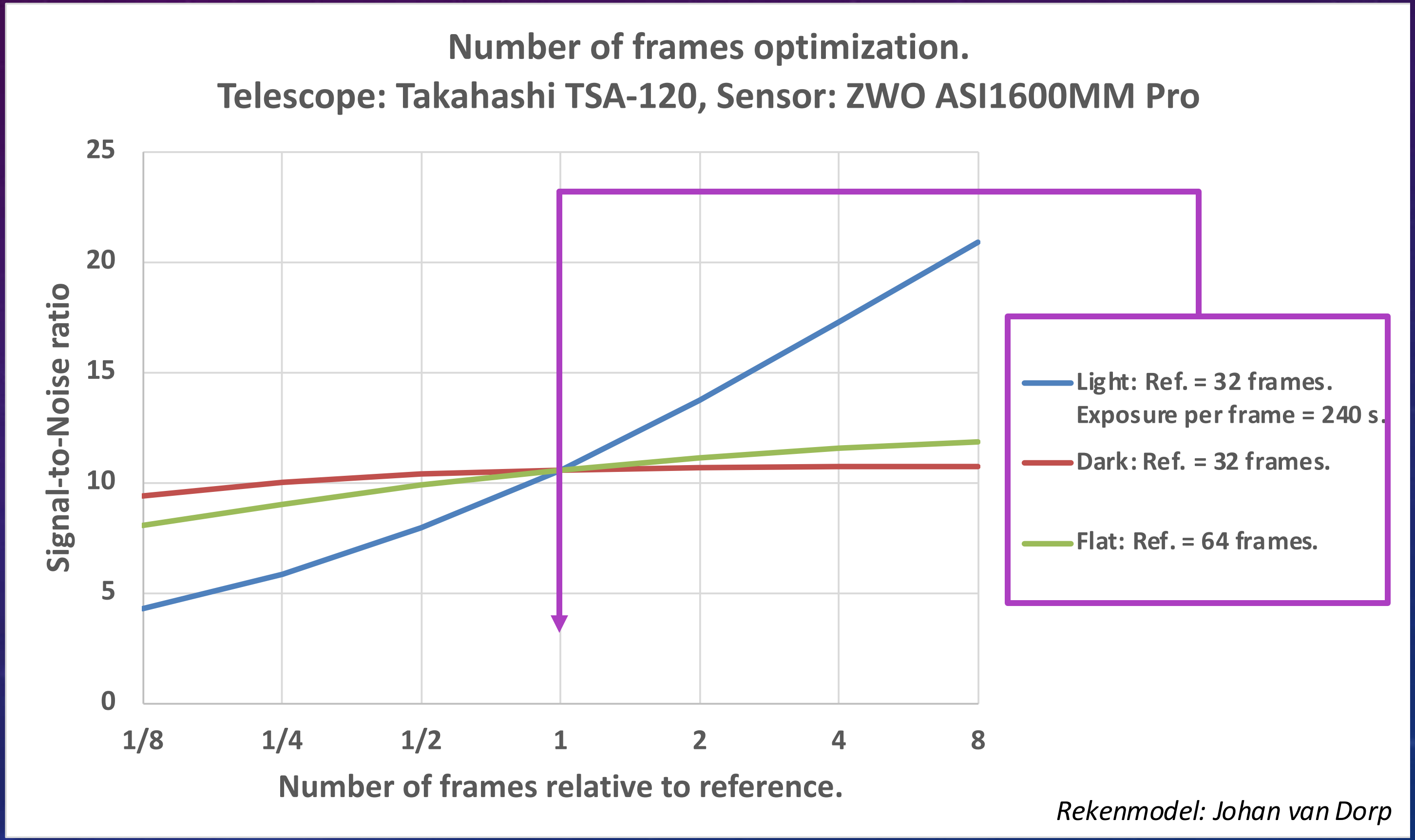
6.4 h (x = 96) → 10x betere S/N

$$\left.\begin{aligned} Signal\colon\ & S_{stack} = x \cdot S_{single} \\ Noise\colon\ & N_{stack}^2 = x \cdot N_{single}^2 \end{aligned}\right\} \quad Signal - to - Noise\colon\ \left({}^{S}/_{N}\right)_{stack} = \sqrt{x} \cdot \left({}^{S}/_{N}\right)_{single}$$

VOLDOENDE *DARK*- EN *FLAT* FRAMES

- Het aantal *Dark*- of *Flat* frames wordt gevarieerd rond de referentiewaarde.

- Conclusie:
  - *De grootste winst valt te halen door meer Light Frames.*
  - *Het aantal Dark- en Flat frames is hier voldoende.*



Rekenmodel: Johan van Dorp

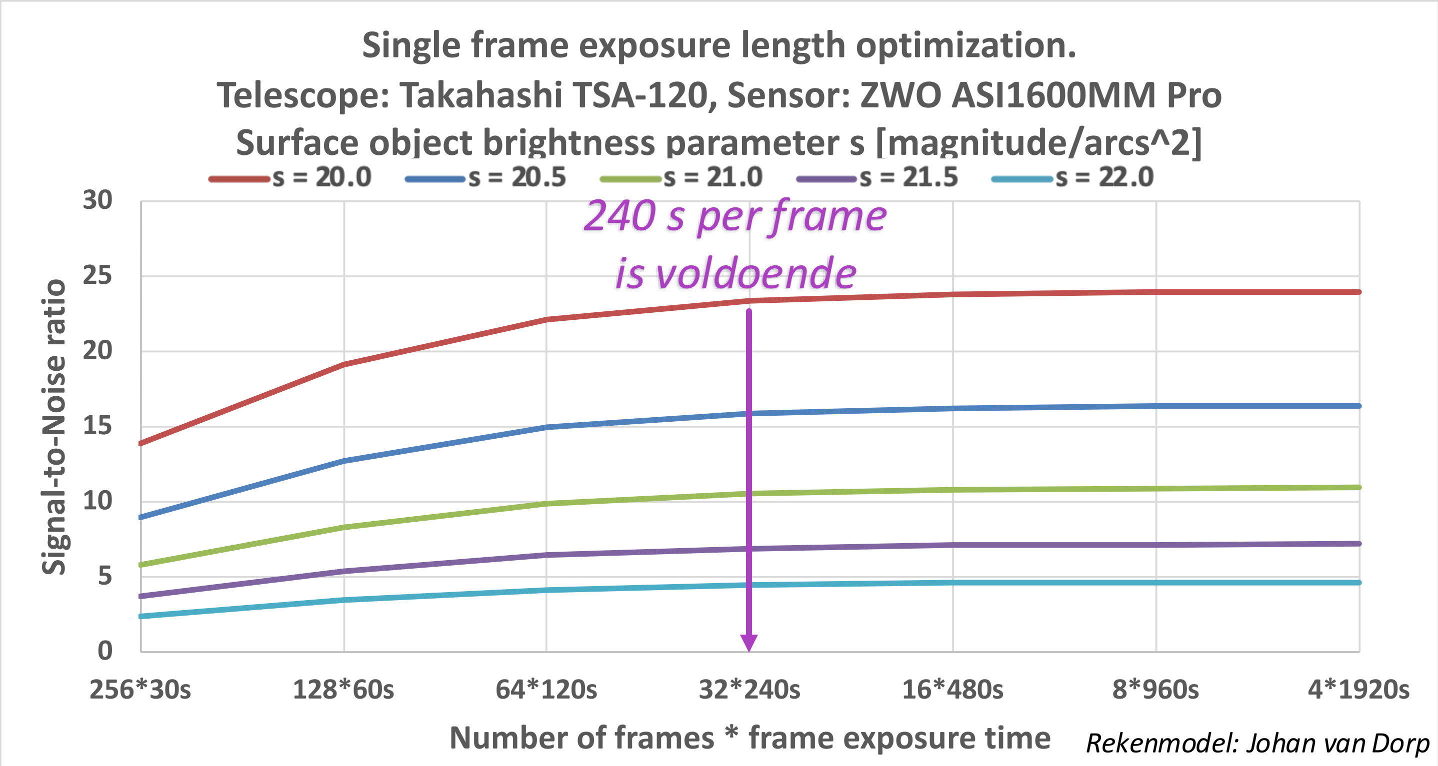
S/N verhouding van zwakke flank van een sterrenstelsel (magnitude 22.5/arcsec^2) met dezelfde belichtingstijd.

240 s BELICHTING PER FRAME IS OPTIMAAL (ASI1600)


23 x 120 s

12 x 240 s

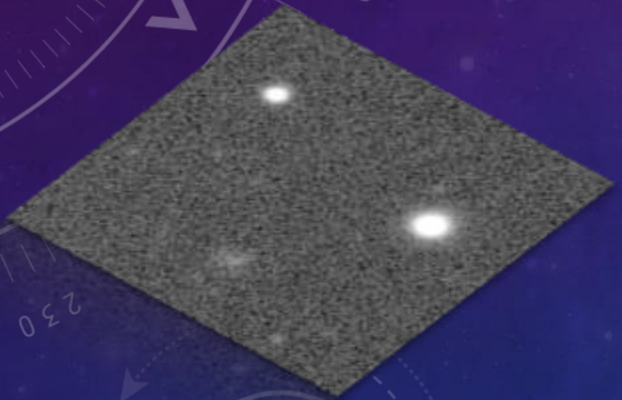
- Uitleesruis is lager bij minder opnamen, langer belicht.
- 2x zoveel opnamen levert 1 *bit* extra dynamiek.
- Meer opnamen goed voor *dithering* en *filteren* van artefacten als satellieten.



240 s per frame is voldoende

Rekenmodel: Johan van Dorp

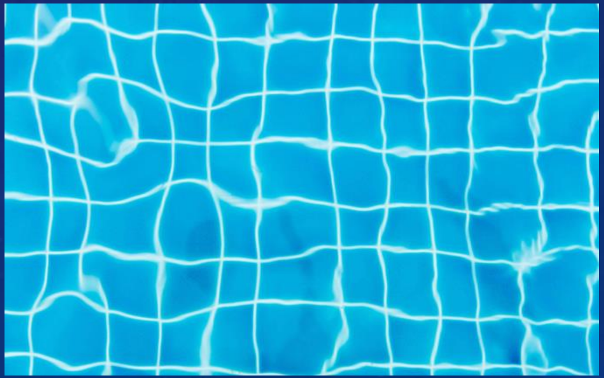
Totale belichtingstijd hetzelfde (7680 s)



GRAVITATIE EN DE RING

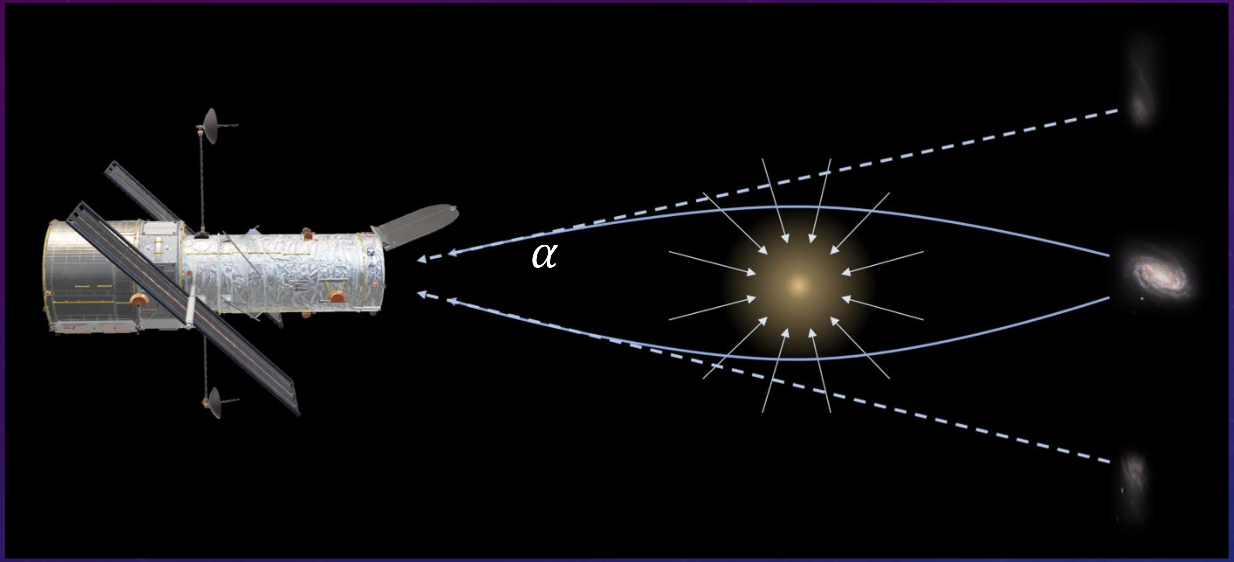
ACTUEEL: EUCLID

- 1.2 meter telescoop op L2 locatie
- Gelanceerd in juli 2023
- Waarom dijt het heelal versneld uit?
- Wat de bron is van donkere materie en energy?
- Effect van gravitatie lenswerking
- Vervorming van de ruimte-tijd:



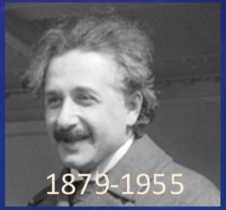
Foto's: ESA

GRAVITATIE EN DE RING



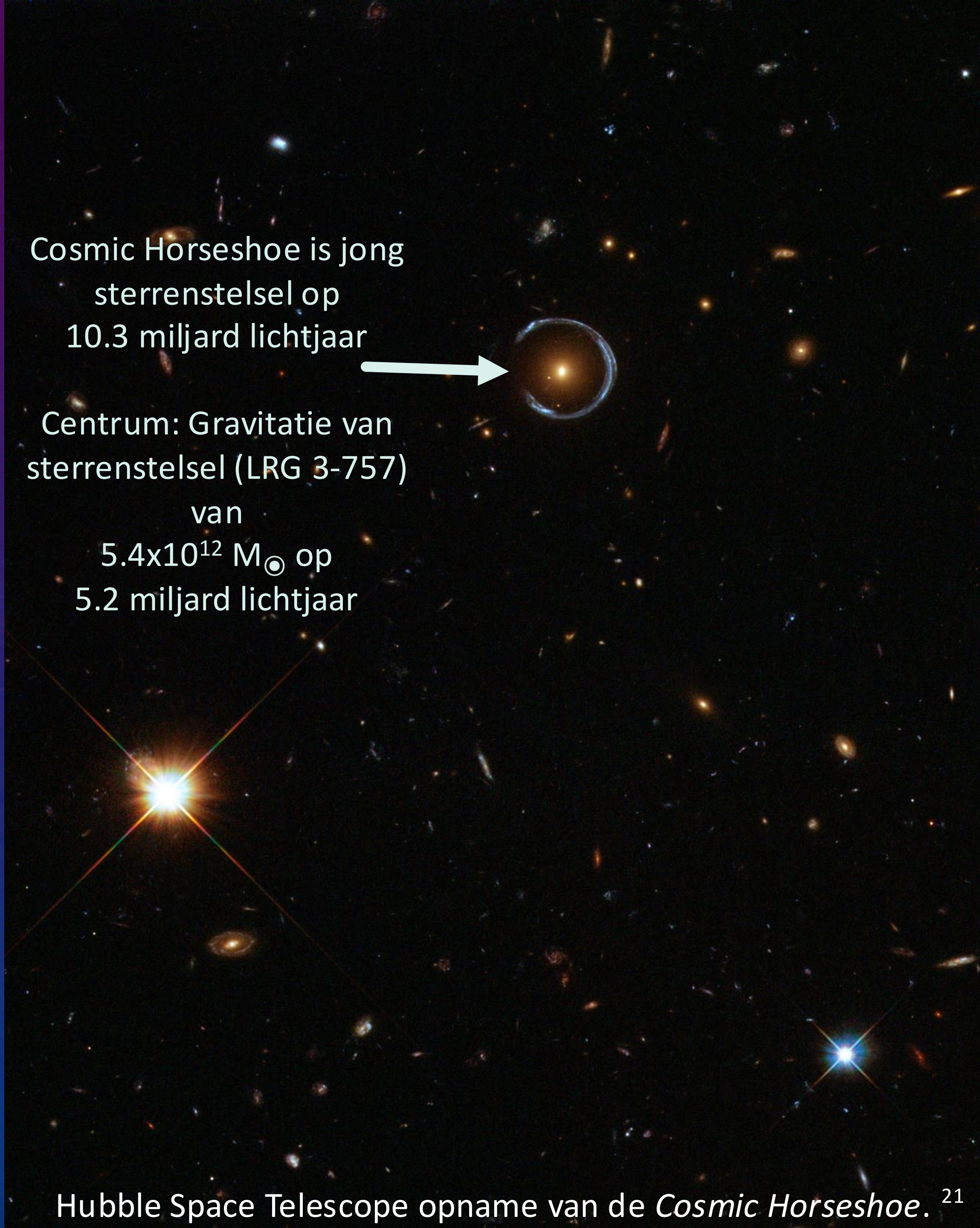
Buiging van licht door vervorming van ruimte en tijd (Einstein)

Grote massa [M] bij invalsafstand [b]: $\alpha = \frac{4G}{c^2}\frac{M}{b}$


1879-1955

G = *Gravitatie constante* = $6.6743 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$

c = *Lichtsnelheid in vacuum* = 299792458 m/s

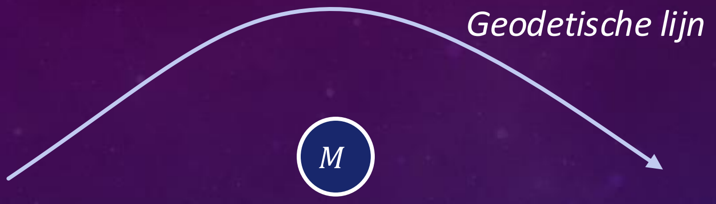


Cosmic Horseshoe is jong sterrenstelsel op 10.3 miljard lichtjaar

Centrum: Gravitatie van sterrenstelsel (LRG 3-757) van $5.4\text{x}10^{12}\ M_{\odot}$ op 5.2 miljard lichtjaar

Hubble Space Telescope opname van de *Cosmic Horseshoe*.

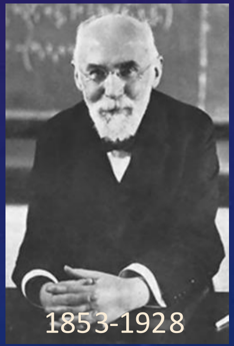
GRAVITATIE



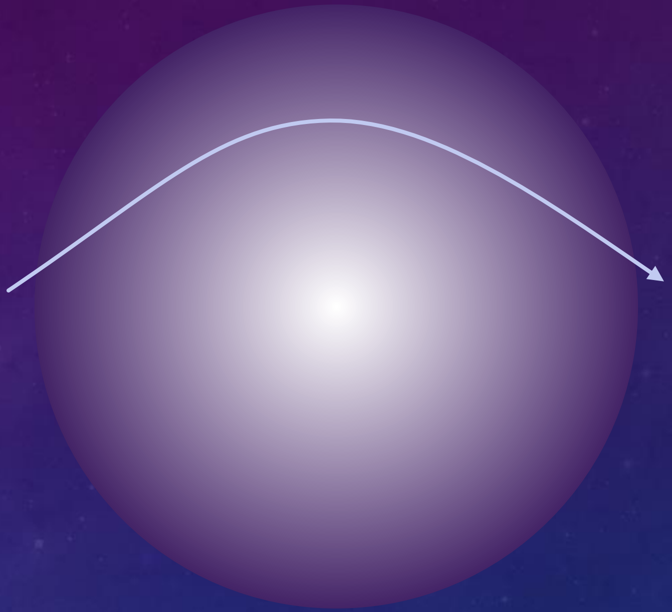
Relativiteit (*lengtecontractie* en *tijddilatatie*) bij snelheid v komt overeen met de buiging van de *Ruimtetijd* door de gravitatie van een massa M op afstand r:

Lorentz-factor:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}$$



1853-1928



Principe van Fermat:

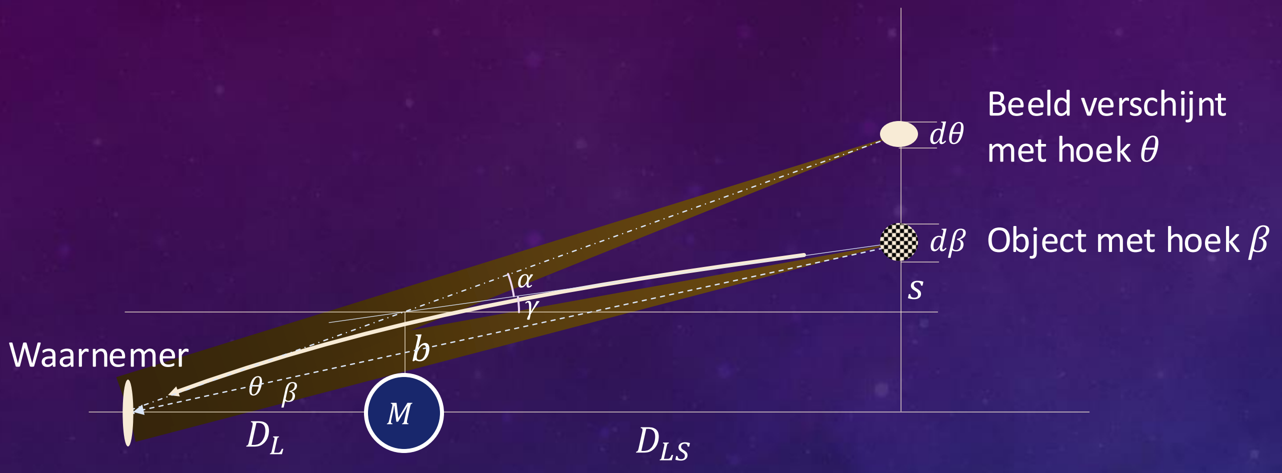
De weg die een lichtstraal tussen twee punten aflegt, is die welke in de kortste tijd afgelegd wordt.

Dit geldt bij media waar de lichtsnelheid veranderd (lenzen) maar ook voor ruimtekromming door een zware massa:

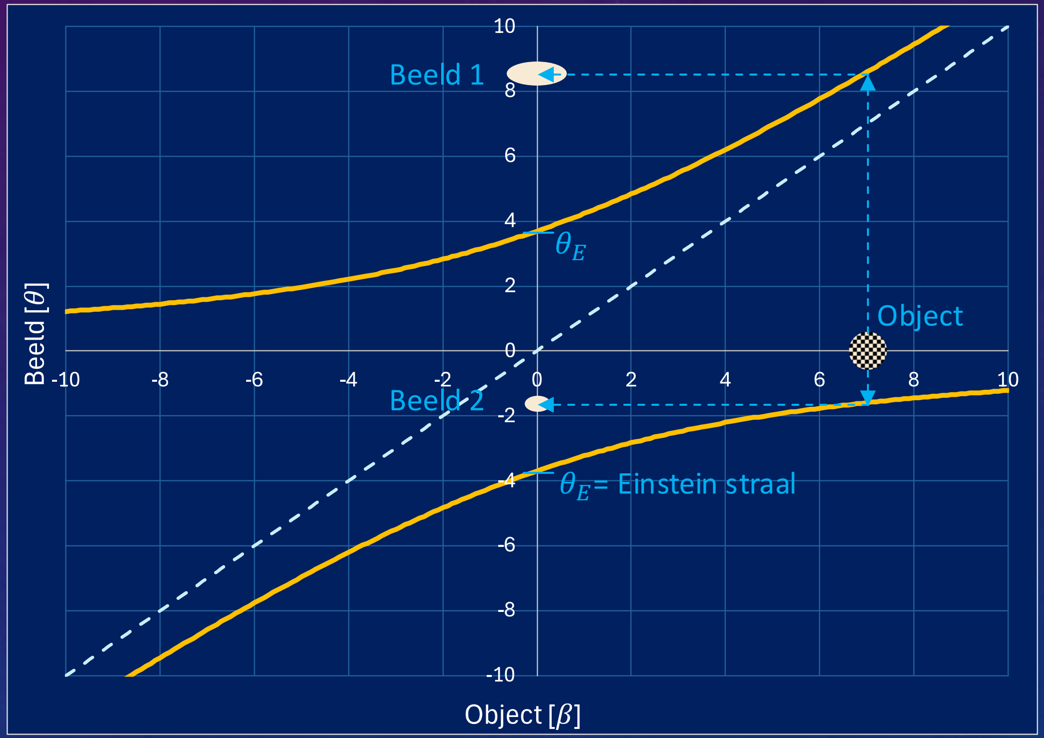
“Gravitational lensing“

1601-1665

LICHTAFBUIGING DOOR GRAVITATIE



$$\beta = \theta - \frac{\theta_E^2}{\theta}$$



Een object dat op een hoek van β, staat wordt waargenomen met een hoek θ omdat de massa M het licht met een hoek α afbuigt:

$$\alpha = \frac{4G}{c^2}\frac{M}{b}$$

Als het object precies rechtdoor staat ($\beta = 0$) vormt het beeld in een ring met de Einstein straal: *de Cosmic Horseshoe*.

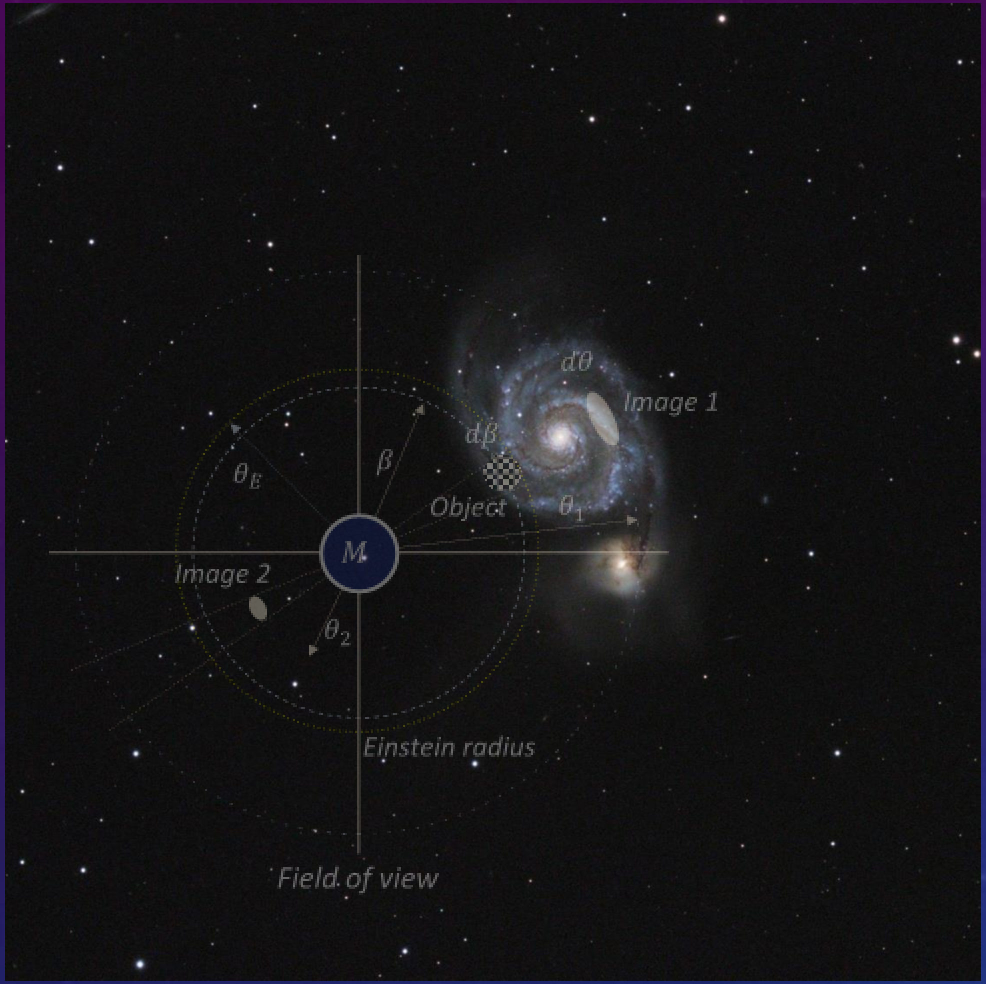
$$\theta_E = 8.8E20\ m\ of\ 3.7''$$

Niet te verwarren met de Schwarzschildstraal waar de ontsnappingssnelheid gelijk aan de lichtsnelheid is:

$$r_s = \frac{2GM}{c^2} = 1.6E16\ m\ of\ 0.000067''$$

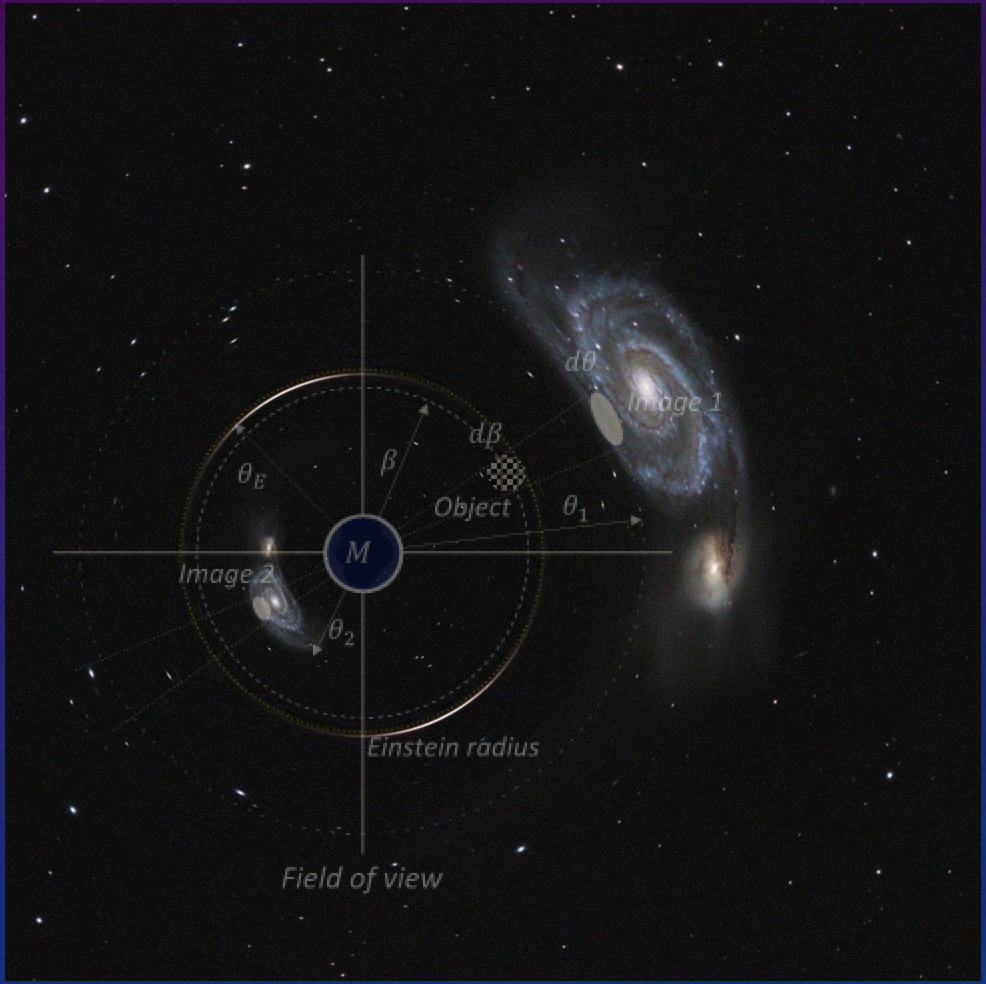
Simulatie van een zware massa dat voor M51 beweegt.

Achtergrond Object, M51, volkomen niet op schaal!



Simulatie

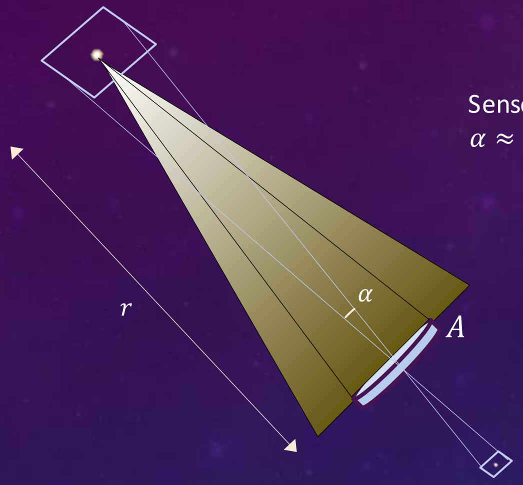
Waargenomen beeld



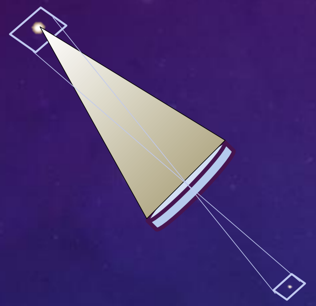
VERSTERKT EEN GRAVITATIELENS?

Referentie: Koen Kuijken

Ster (puntbron)



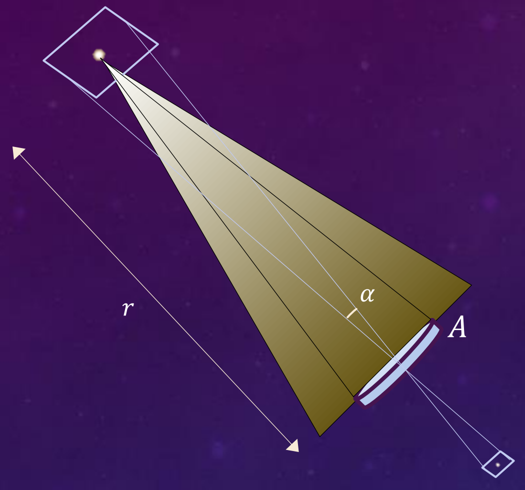
Sensor pixel scale
$\alpha \approx \tan\alpha = p/f$



De hoeveelheid sterlicht gedetecteerd door een sensor is onafhankelijk van de ruimtehoek α.

Een ster dichterbij schijnt helderder met $\frac{1}{r^2}$.

Ster (puntbron)



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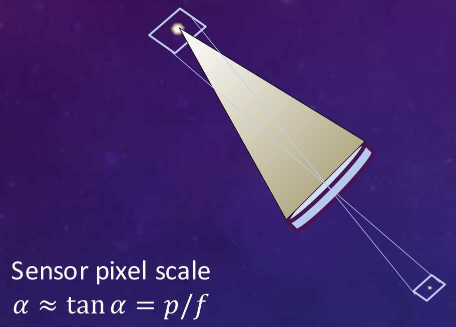
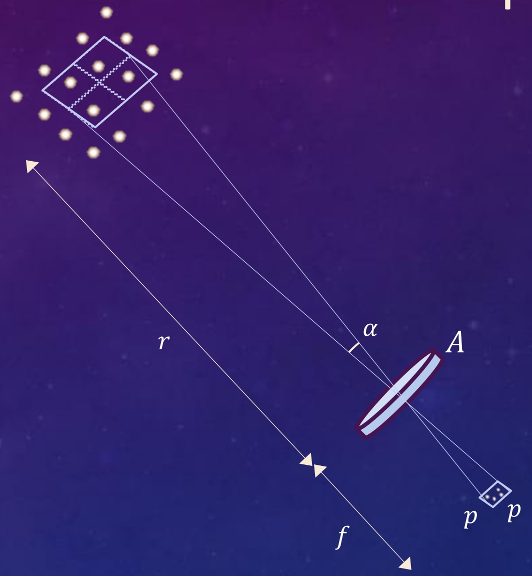


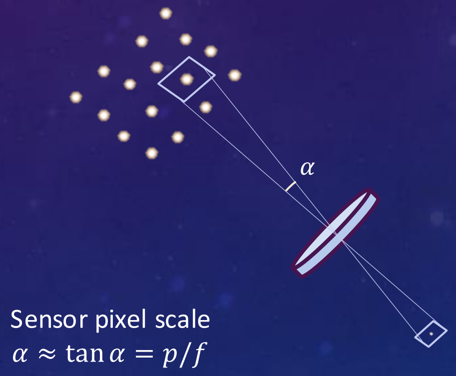
Sensor pixel scale
$\alpha \approx \tan\alpha = p/f$

Een ster dichterbij schijnt helderder met $\frac{1}{r^2}$.

Een sterrenstelsel heeft een oppervlaktehelderheid, zeg: 1 ster per pixel



Een grotere pixel, vangt meer licht van de nevel.

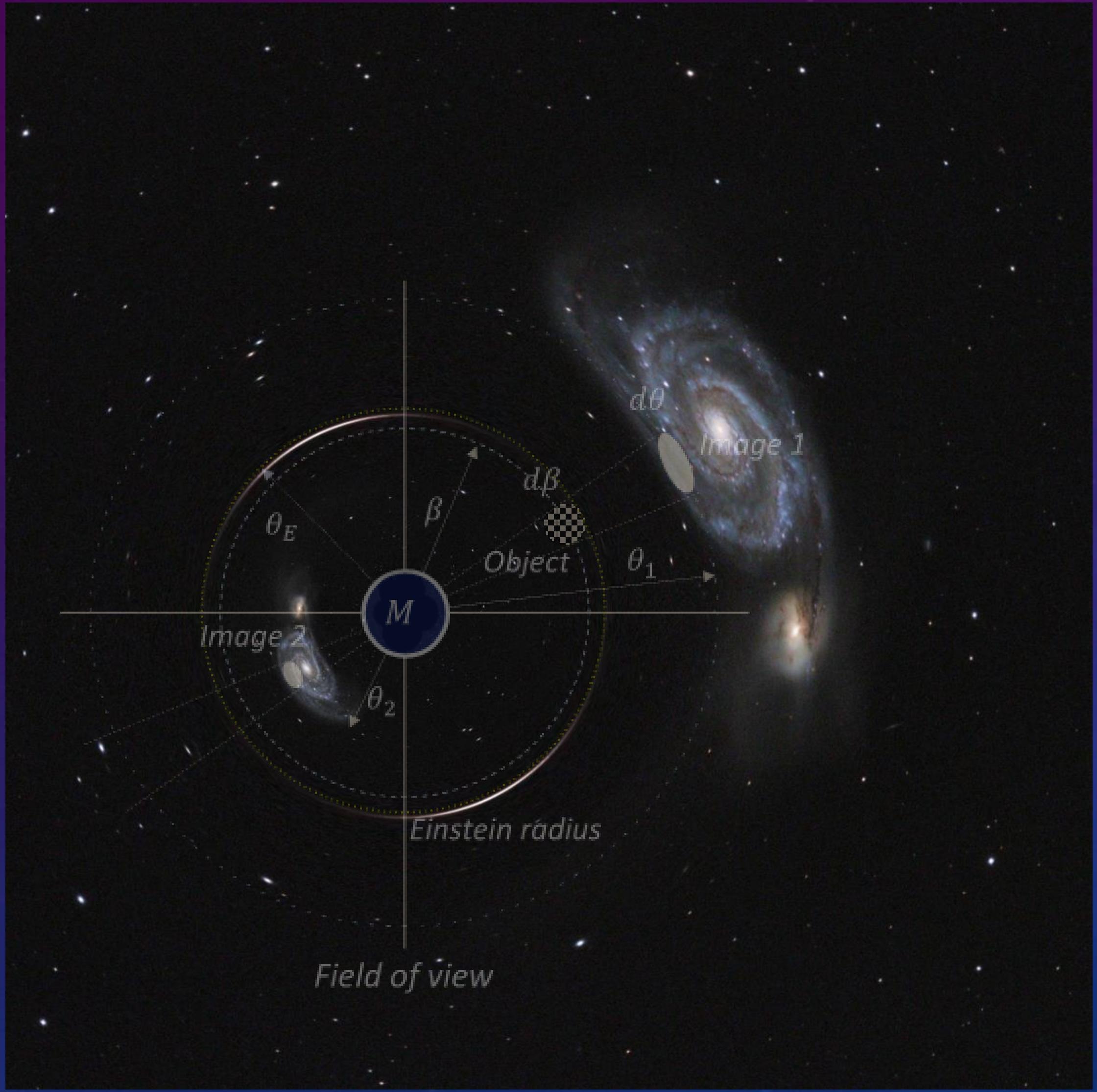


Sensor pixel scale
$\alpha \approx \tan\alpha = p/f$

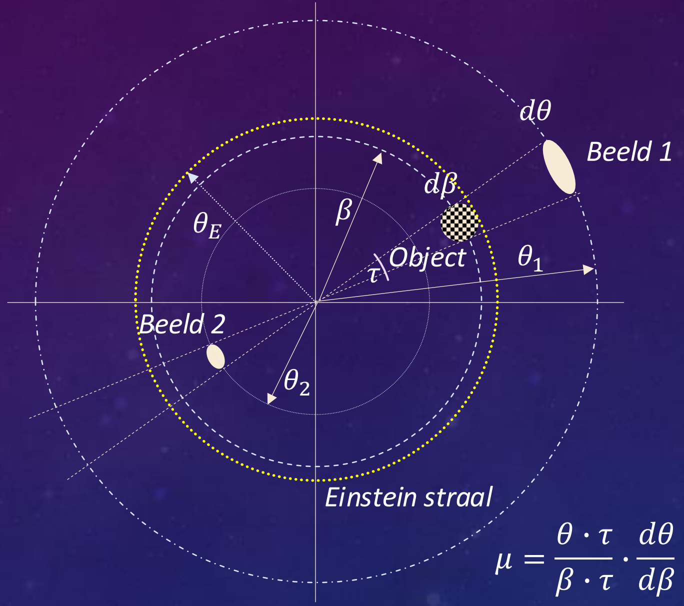
Is de nevel dichterbij, dan is het gebied per pixel kleiner. Dat gebied schijnt wel helderder.

De oppervlaktehelderheid veranderd niet voor de waarnemer!
Ook niet als de lichtweg wordt vervormd.

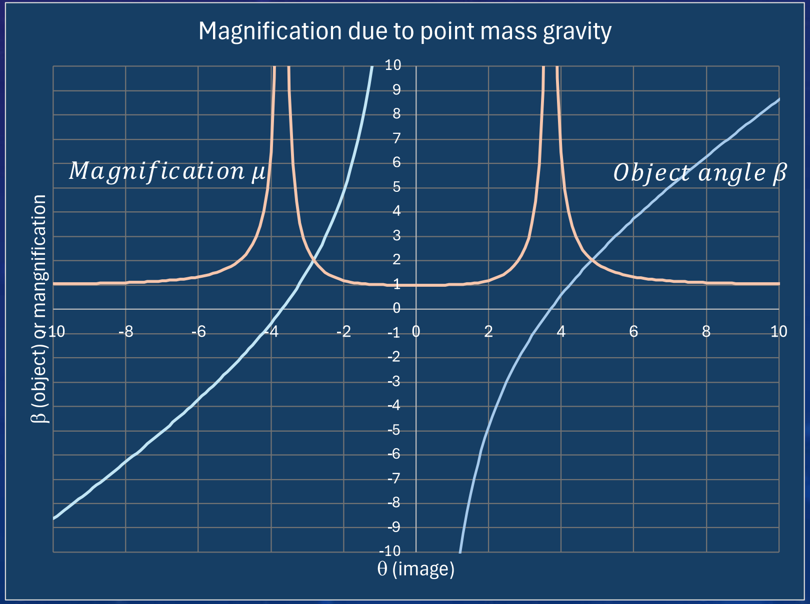
Ster (puntbron)



Een ster schijnt als puntbron maar de *totale* hoeveelheid licht wordt groter omdat het sterlicht wordt uitgerekt tot een ring bij een objecthoek $\beta = 0$.

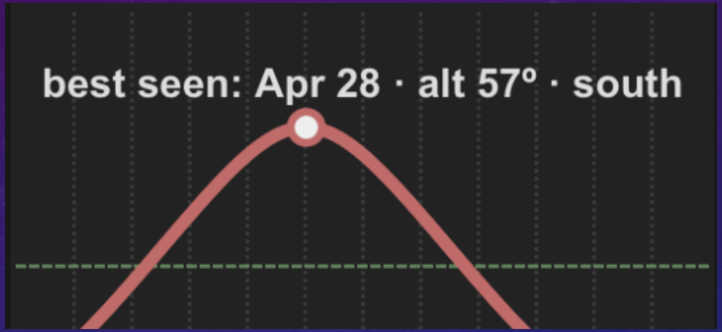


$$\mu = \frac{\theta \cdot \tau}{\beta \cdot \tau} \cdot \frac{d\theta}{d\beta}$$



WAAR STAAT DE *COSMIC HORSEHOE*?

Sterrenbeeld Leeuw



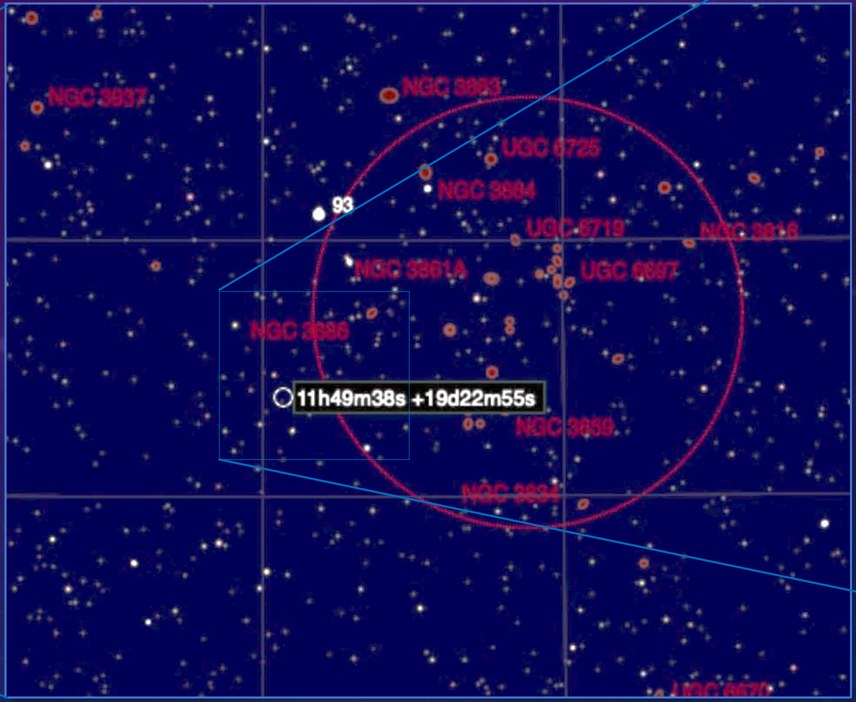
Zichtbaarheid van Cosmic Horseshoe in Nederland 22:00.

WAAR STAAT DE *COSMIC HORSEHOE*?

Sterrenbeeld Leeuw

WAAR STAAT DE *COSMIC HORSEHOE*?

Sterrenbeeld Leeuw

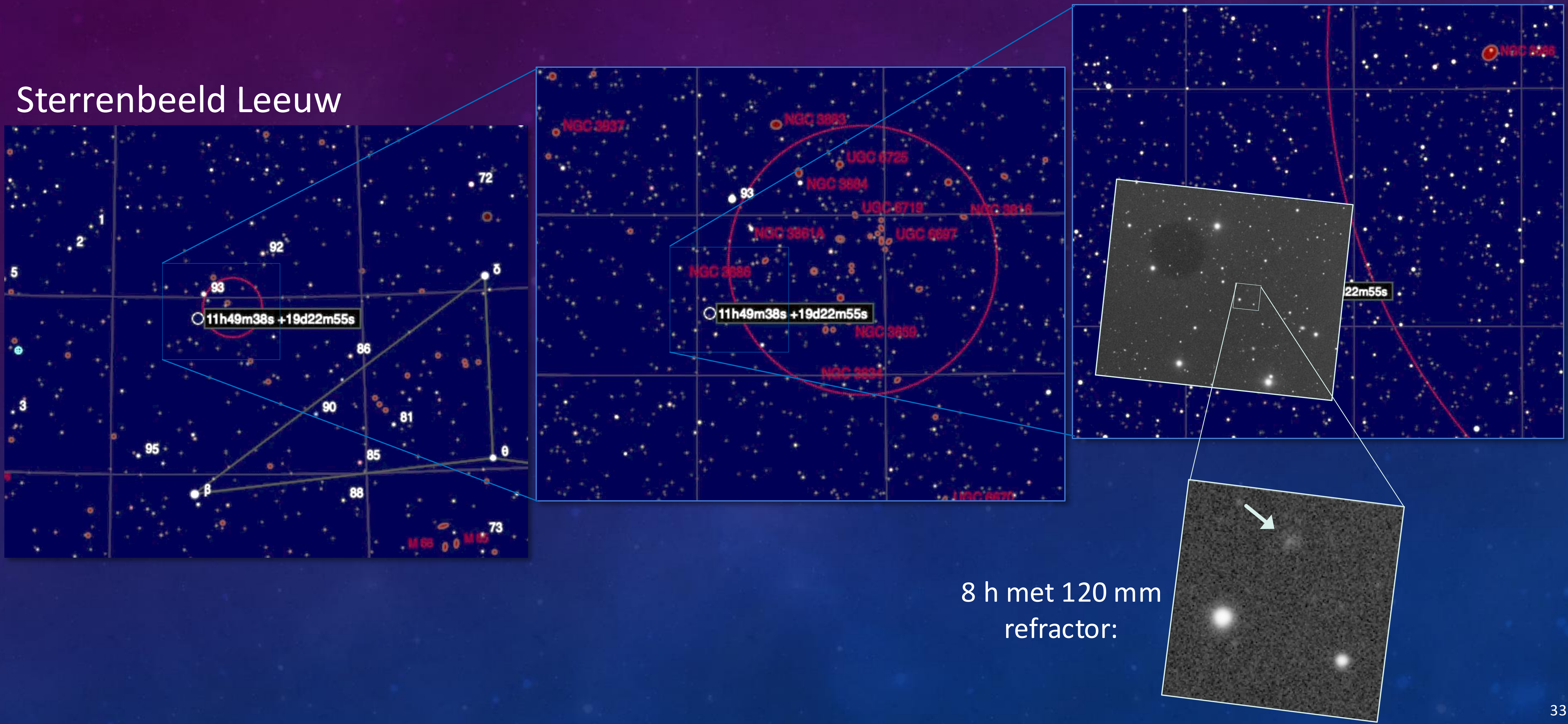


11h 49m 38s +19d 22m 55s

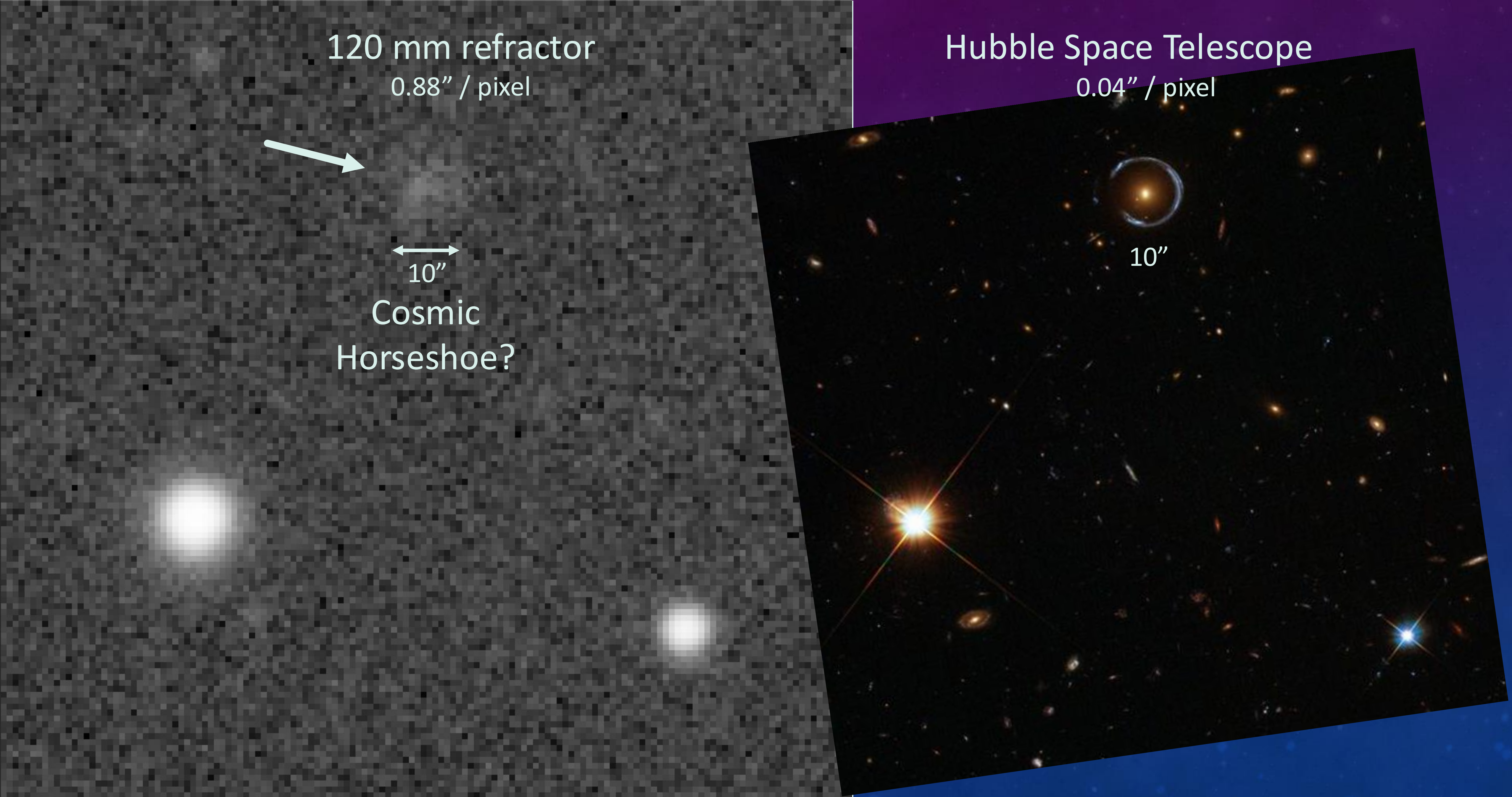
WAAR STAAT DE *COSMIC HORSEHOE*?

De Cosmic Horseshoe is magnitude 20.3 (roodverschuiving z = 2.379)

Sterrenbeeld Leeuw



8 h met 120 mm refractor:



120 mm refractor

0.88” / pixel

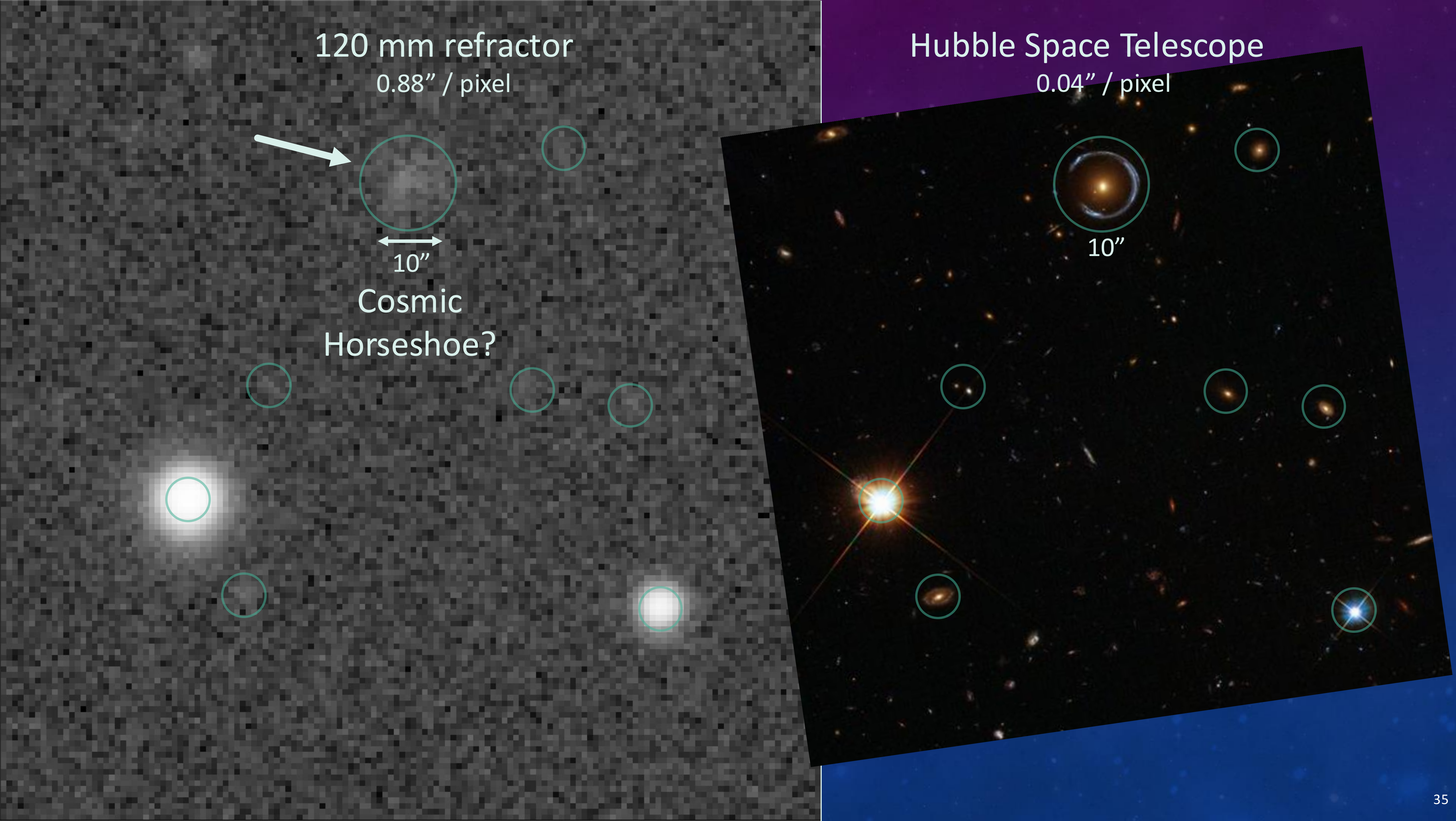
10”

Cosmic Horseshoe?

Hubble Space Telescope

0.04” / pixel

10”



120 mm refractor

0.88” / pixel

Hubble Space Telescope

0.04” / pixel

120 mm refractor

0.88” / pixel

Cosmic Horseshoe?

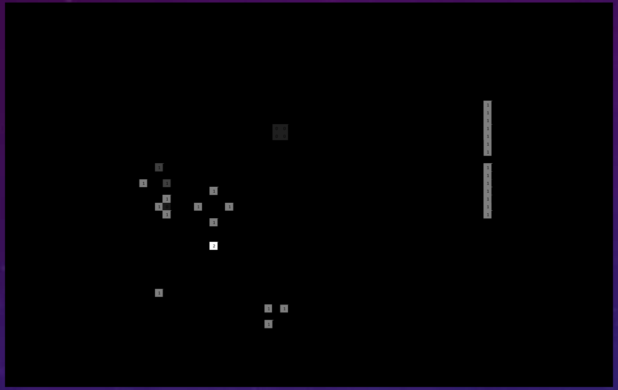
Kunnen we dit beeld van de ring scherper maken met **deconvolutie**?

Hubble Space Telescope

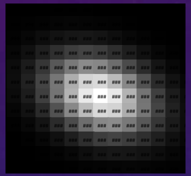
0.04” / pixel

WAT IS DECONVOLUTIE?

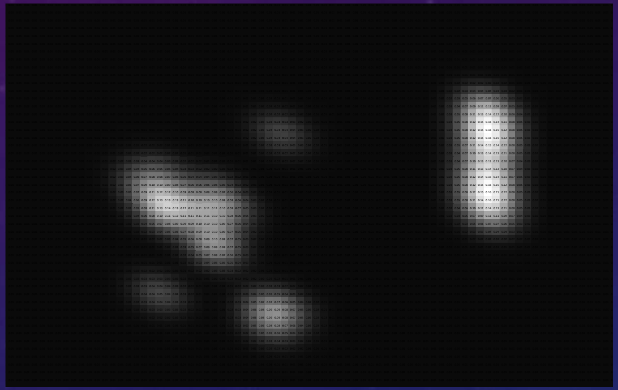
Convolutie:



Object $f(x,y)$



Dispersie $g(i,j)$



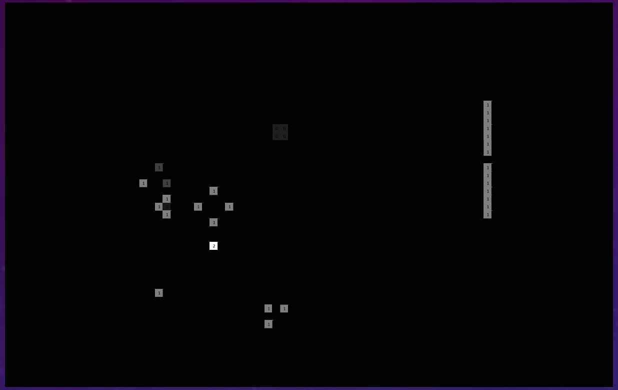
Beeld $d(x,y) = f * g$

Dispersie, bijvoorbeeld:

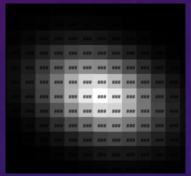
- *Onscherpte door atmosfeer*
- *Onscherpte door optiek*
- *Bewogen beeld*
- *Willekeurige andere vervormingen*

WAT IS DECONVOLUTIE?

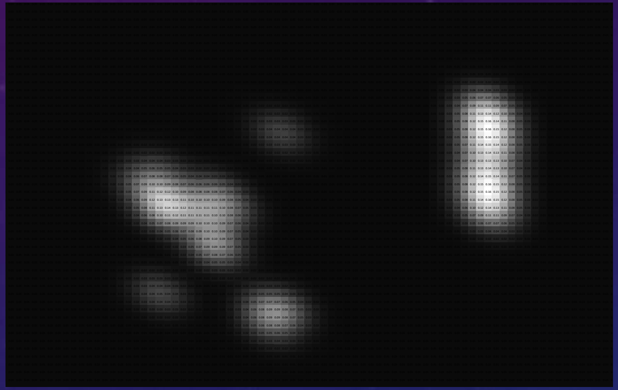
Convolutie:



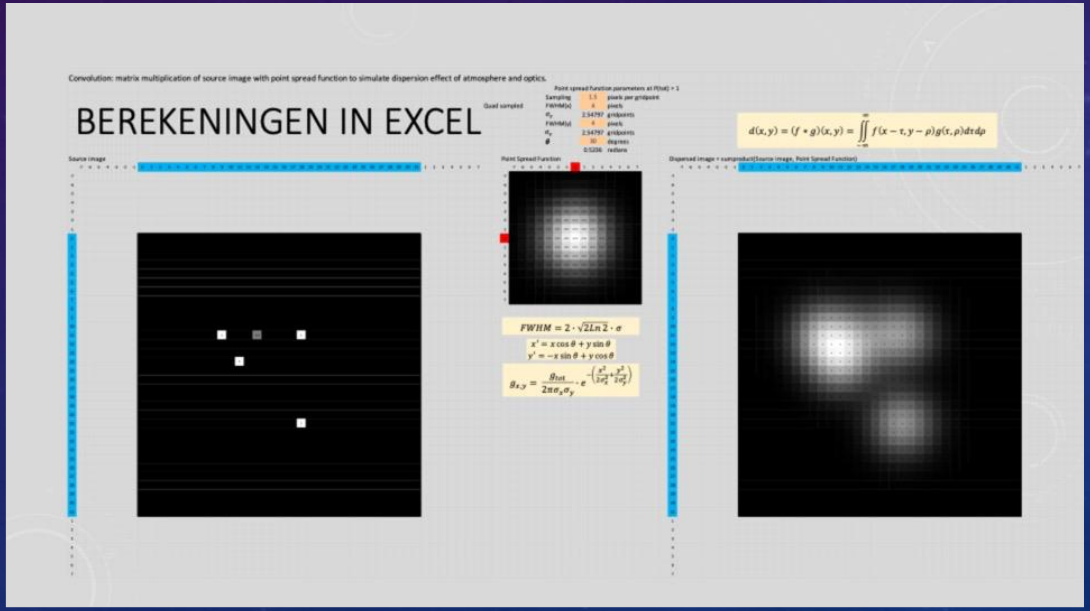
$Object\ f(x,y)$



$Dispersie\ g(i,j)$



$Beeld\ d(x,y) = f * g$



Convolution: matrix multiplication of source image with point spread function to simulate dispersion effect of atmosphere and optics.

BEREKENINGEN IN EXCEL

$$d(x,y) = (f * g)(x,y) = \iint_{-\infty}^{\infty} f(x-\tau, y-\rho)\, g(\tau,\rho)\, d\tau\, d\rho$$

$$FWHM = 2 \cdot \sqrt{2 Ln 2} \cdot \sigma$$

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

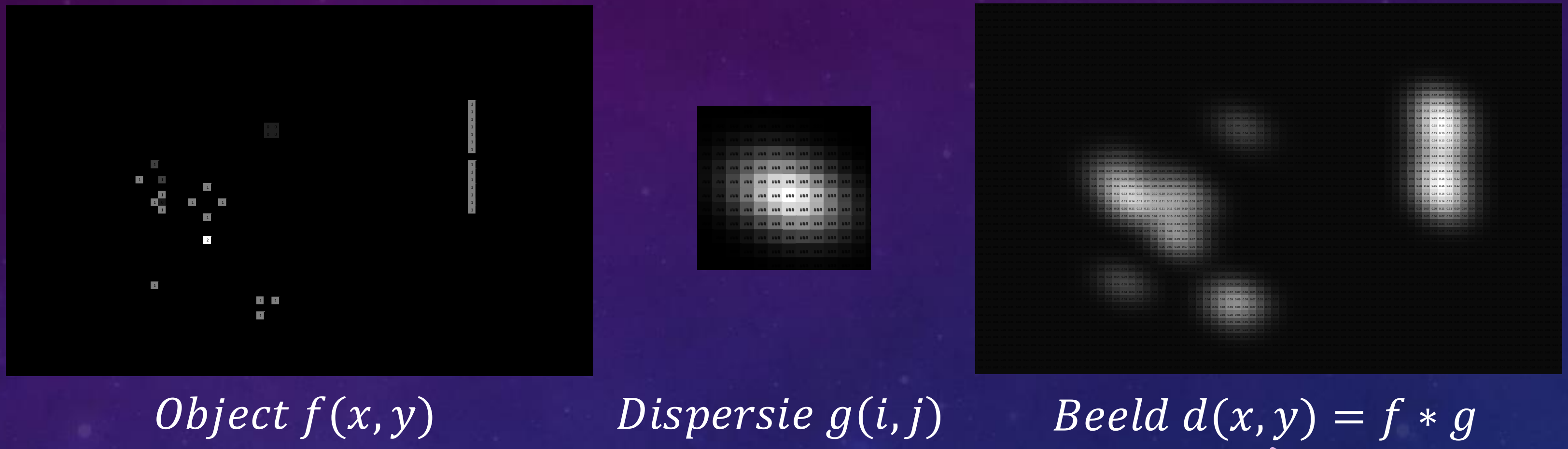
$$g_{x,y} = \frac{g_{tot}}{2\pi\sigma_x\sigma_y} \cdot e^{-\left(\frac{x'^2}{2\sigma_x^2} + \frac{y'^2}{2\sigma_y^2}\right)}$$

Berekeningen in Excel.

Kan ook in bijvoorbeeld Siril en PixInsight,
maar berekeningen zijn dan niet transparant.

WAT IS DECONVOLUTIE?

Convolutie:

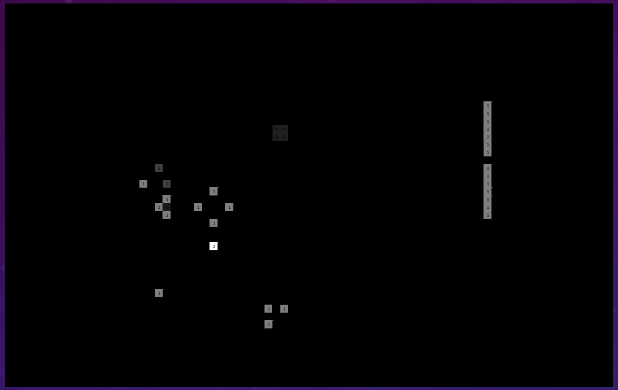


Deconvolutie:

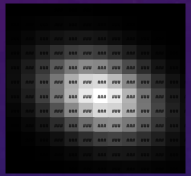
$$\frac{Beeld\ d(x,y) = f(x,y) * g(i,j)}{Dispersie\ g(i,j)} = Object\ f(x,y)$$

WAT IS DECONVOLUTIE?

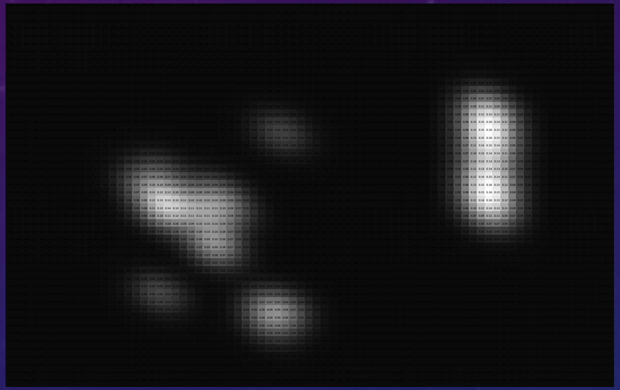
Convolutie:



$Object\ f(x,y)$



$Dispersie\ g(i,j)$



$Beeld\ d(x,y) = f * g$

Zoals een staartdeling iteratief is:

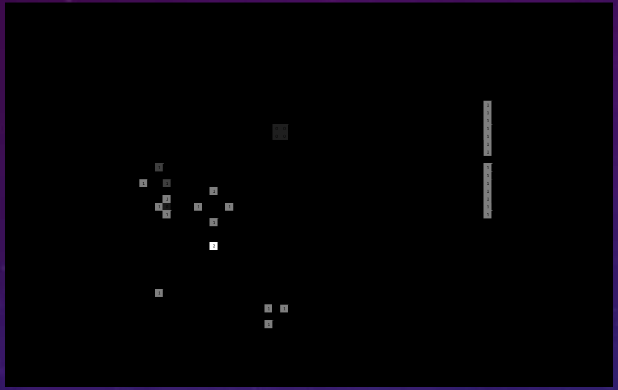
7 / 20.00 \ 2.85…
14
60
56
40
35 …

Deconvolutie:

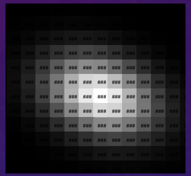
$$\frac{Beeld\ d(x,y) = f(x,y) * g(i,j)}{Dispersie\ g(i,j)} = Object\ f(x,y)$$

WAT IS DECONVOLUTIE?

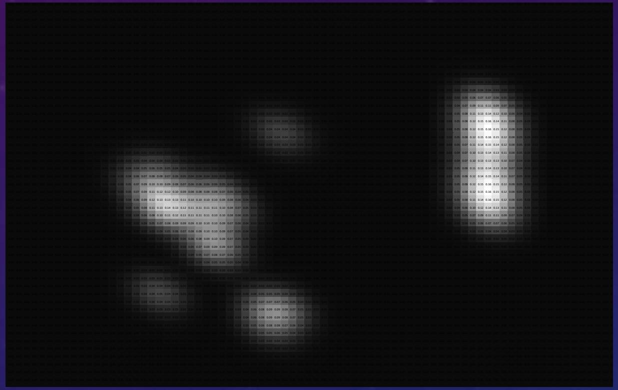
Convolutie:



Object $f(x,y)$

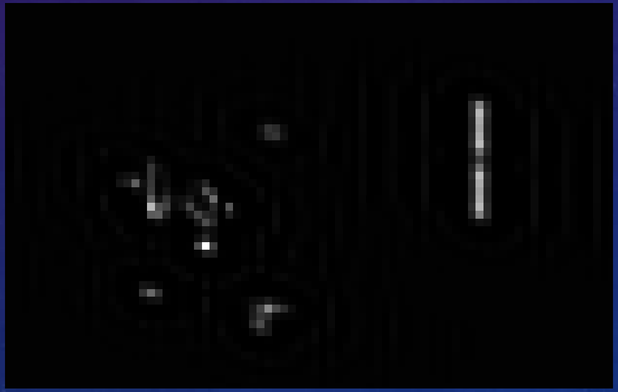


Dispersie $g(i,j)$

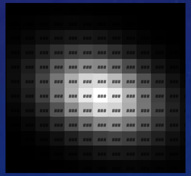


Beeld $d(x,y) = f * g$

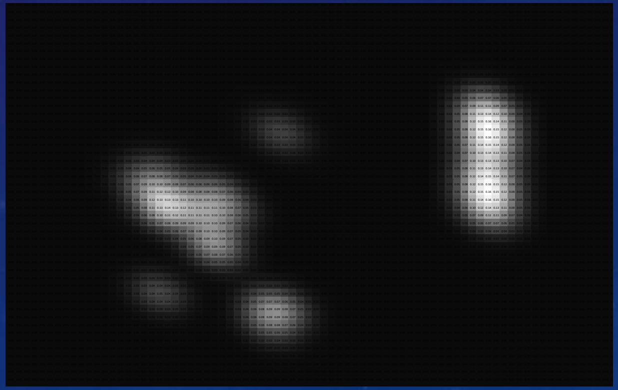
Deconvolutie:



Geschatte *Object* $f'(x,y)$
(na 1000 iteraties)



Dispersie $g'(i,j)$

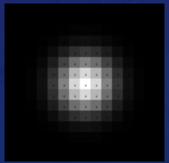


Beeld $d'(x,y) = f' * g'$

VOORBEELD DECONVOLUTIE

(IDEAAL – ZONDER RUIS –IN EXCEL)

Beeld
(gesimuleerde convolutie)



Beeld na deconvolutie
100 iteraties

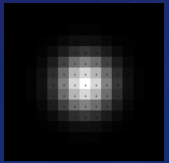
Is dit echt?

Dispersie met *Point-Spread Function* die elk beeldpunt spreidt.

VOORBEELD DECONVOLUTIE

Ringen worden zichtbaar; het werkt!

Beeld
(gesimuleerde convolutie)



Beeld na deconvolutie
100 iteraties

Oorspronkelijke object

Dispersie met *Point-Spread Function* die elk beeldpunt spreidt.

120 mm refractor

0.88” / pixel

Cosmic Horseshoe

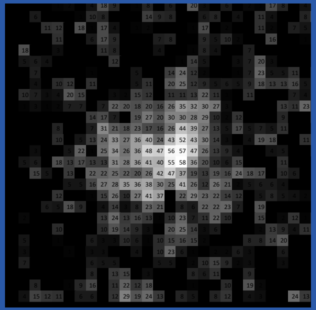
Gestelde vraag:
Kunnen we dit beeld van de ring scherper maken met **deconvolutie**?

Hubble Space Telescope

0.04” / pixel

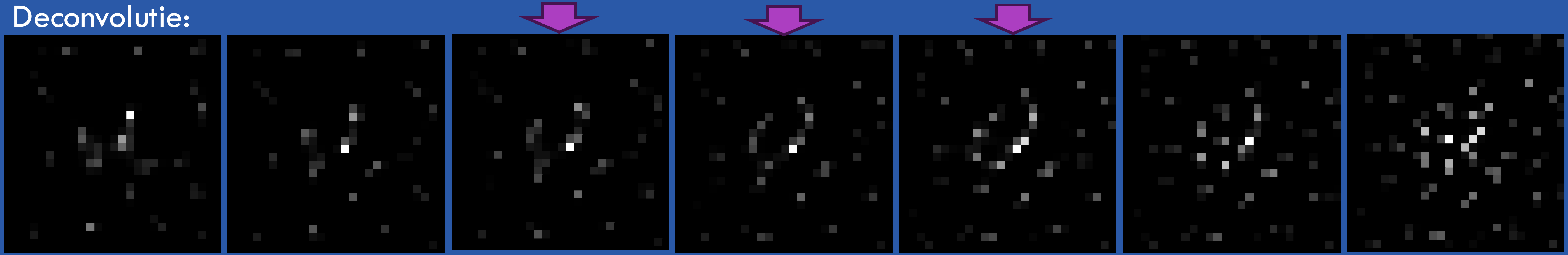
COSMIC HORSESHOE DECONVOLUTIE

Beeld

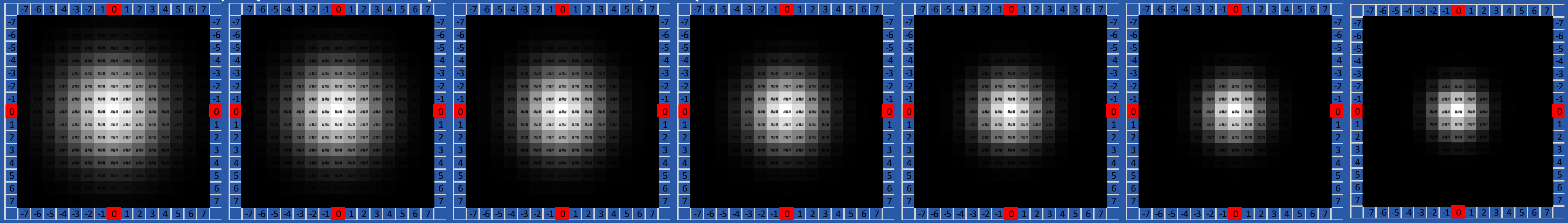


12 cm kijker

Deconvolutie:



1000 Iteraties (RL) met Point-Spread Function (PSF):



FWHM 6.0 pixels

FWHM 5.5 pixels

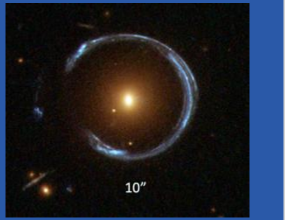
FWHM 5.0 pixels

FWHM 4.5 pixels

FWHM 4.0 pixels

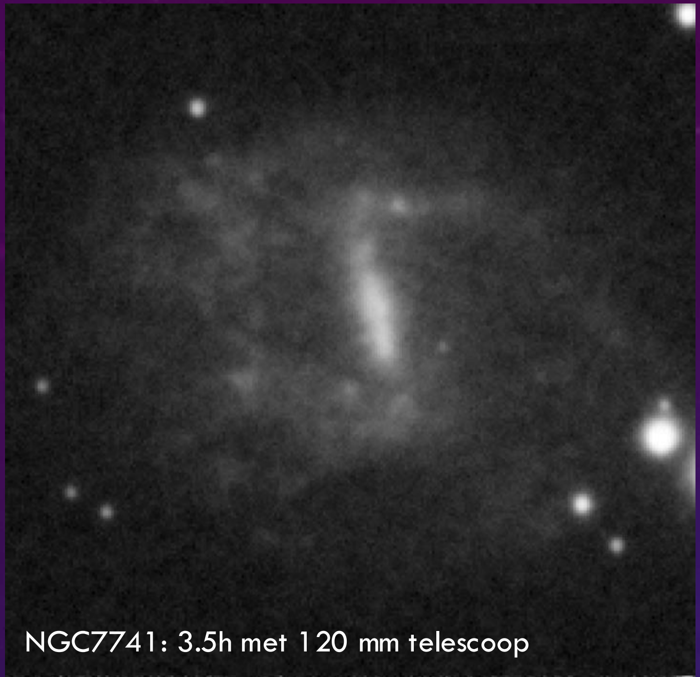
FWHM 3.5 pixels

FWHM 3.0 pixels

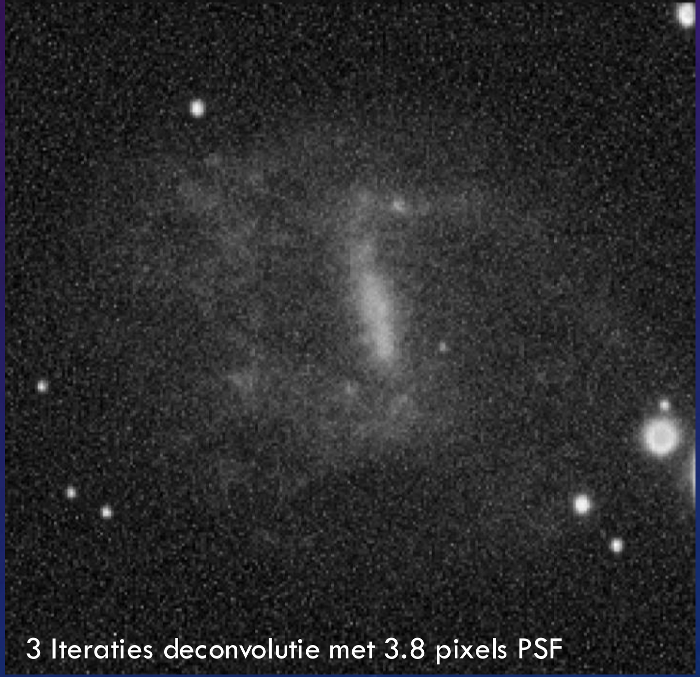


HST
Zelfde schaal

- Een PSF van als verwacht 4-5 pixels (seeing van ~4”) toont iets van een ring.
- De centrale galaxy die de ruimte-tijd vervorming veroorzaakt, is nu scherper.



NGC7741: 3.5h met 120 mm telescoop



3 Iteraties deconvolutie met 3.8 pixels PSF

CONCLUSIE

- *Deconvolutie reconstrueert het waargenomen object – ook bij de Cosmic Horseshoe,*

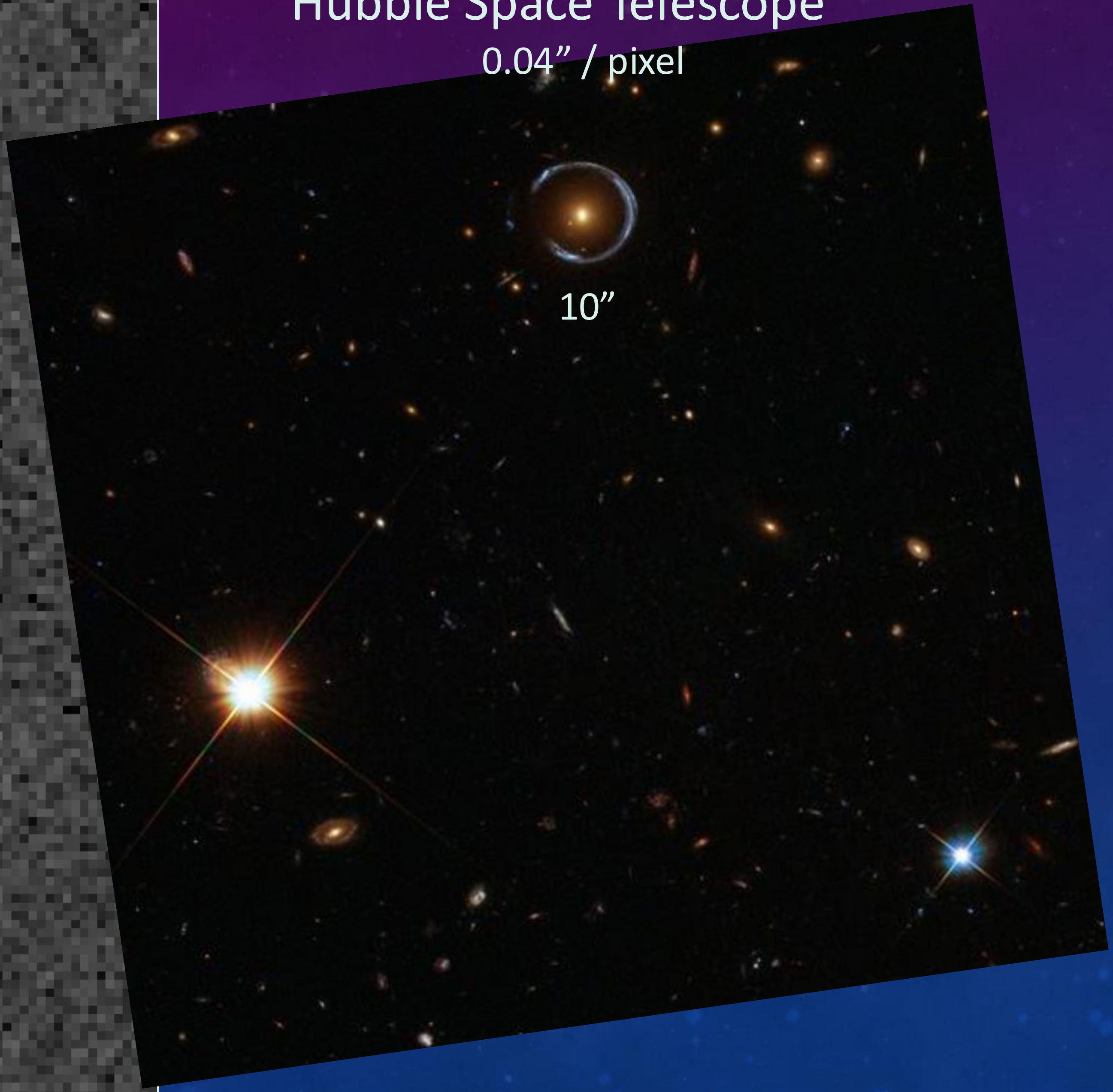
maar…

- *ruis wordt versterkt.*

120 mm refractor

0.88” / pixel

Wat zien we nu van de Cosmic Horse Shoe?

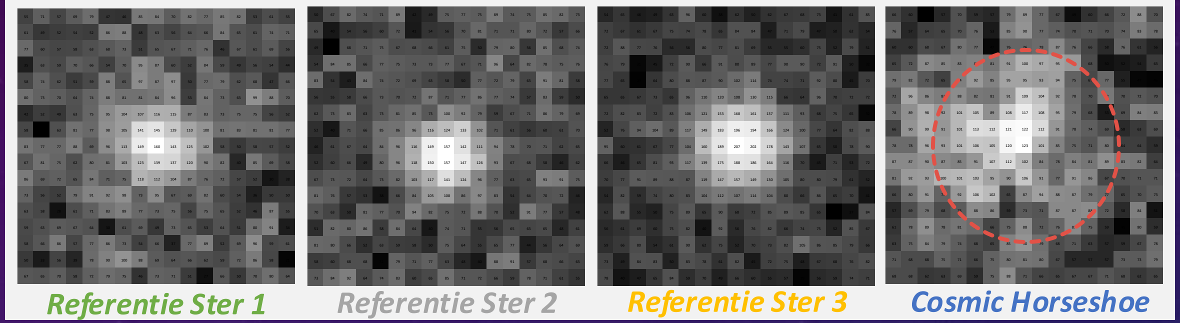


Hubble Space Telescope

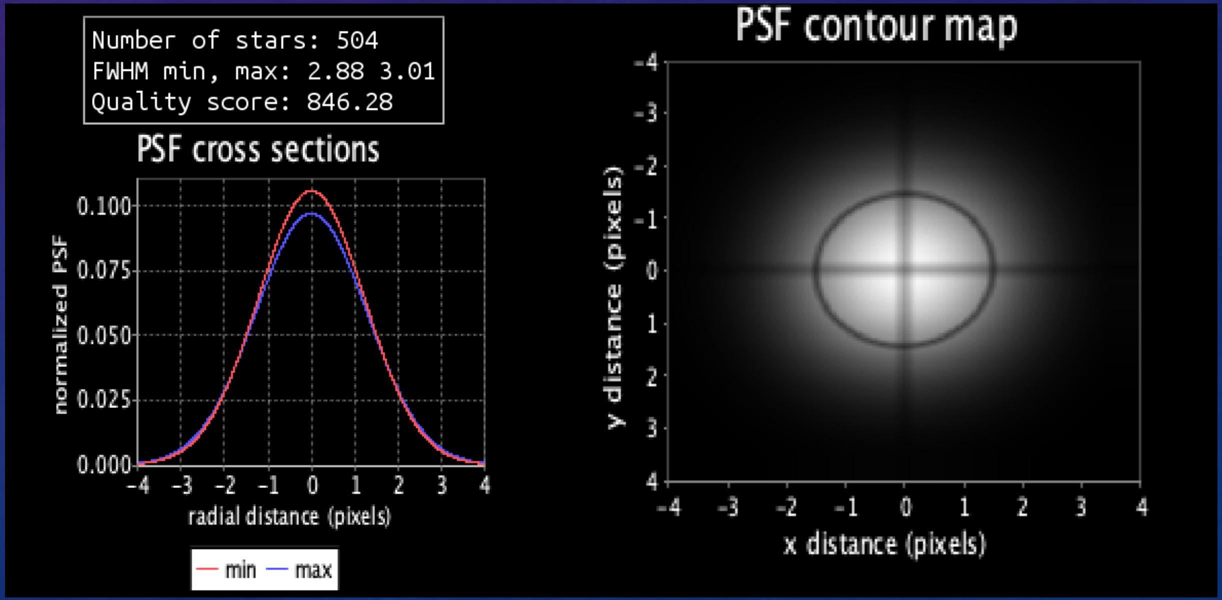
0.04” / pixel

Analyse van Cosmic Horseshoe

- Vergelijk met sterren
- Radiale pixel helderheid
- Gaussische verdeling

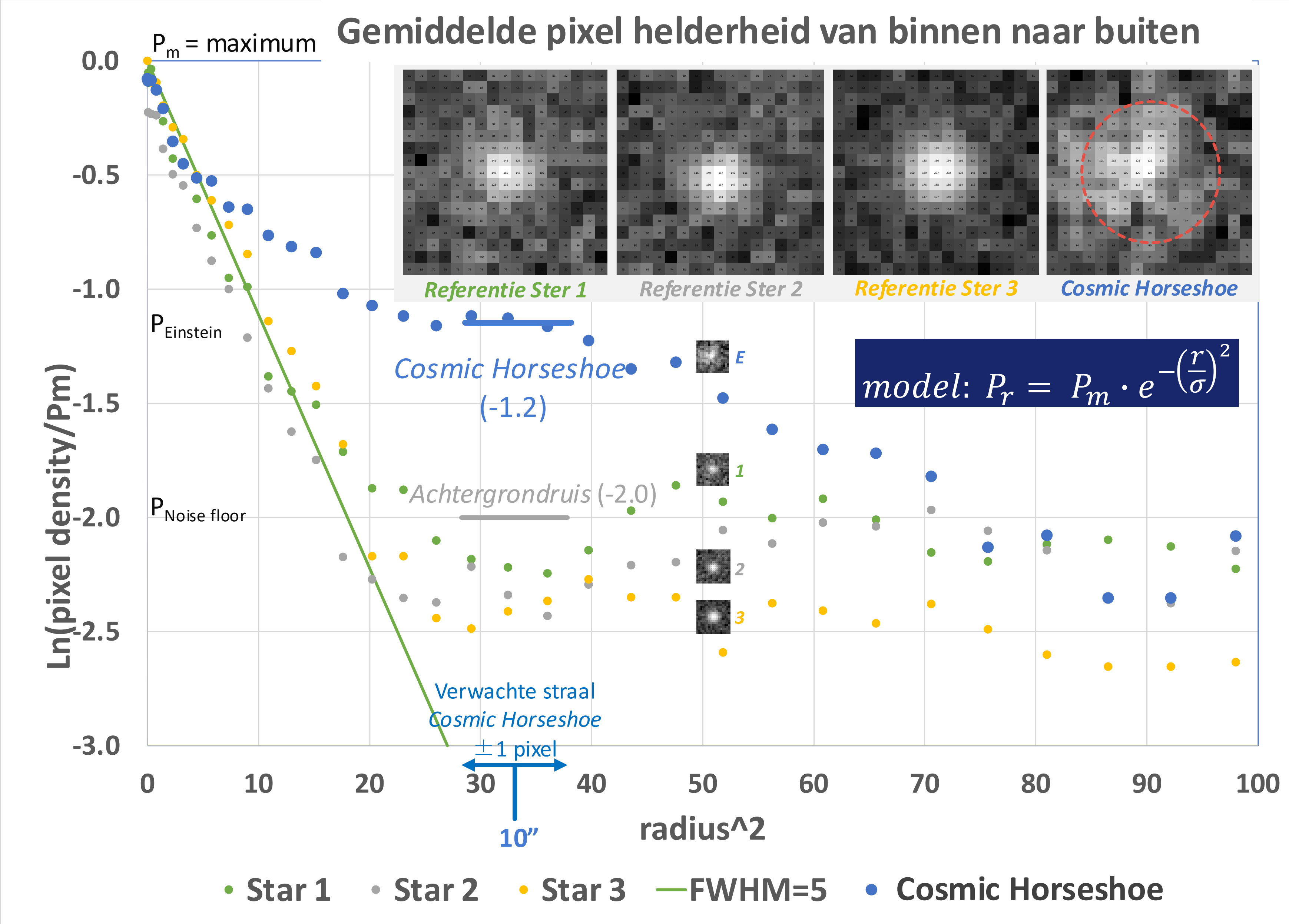

Referentie Ster 1
Referentie Ster 2
Referentie Ster 3
Cosmic Horseshoe

$$model:\ P_r = P_m \cdot e^{-\left(\frac{r}{\sigma}\right)^2}$$



```
Number of stars: 504
FWHM min, max: 2.88 3.01
Quality score: 846.28
```

PSF = Point Spread Function



Cosmic Horseshoe vergeleken met sterlichtspreiding

Uit grafiek:

P_m = 227 elektronen per pixel

$P_{Einstein}$ = 68 elektronen per pixel

$P_{Noise\ floor}$ = 30 elektronen per pixel

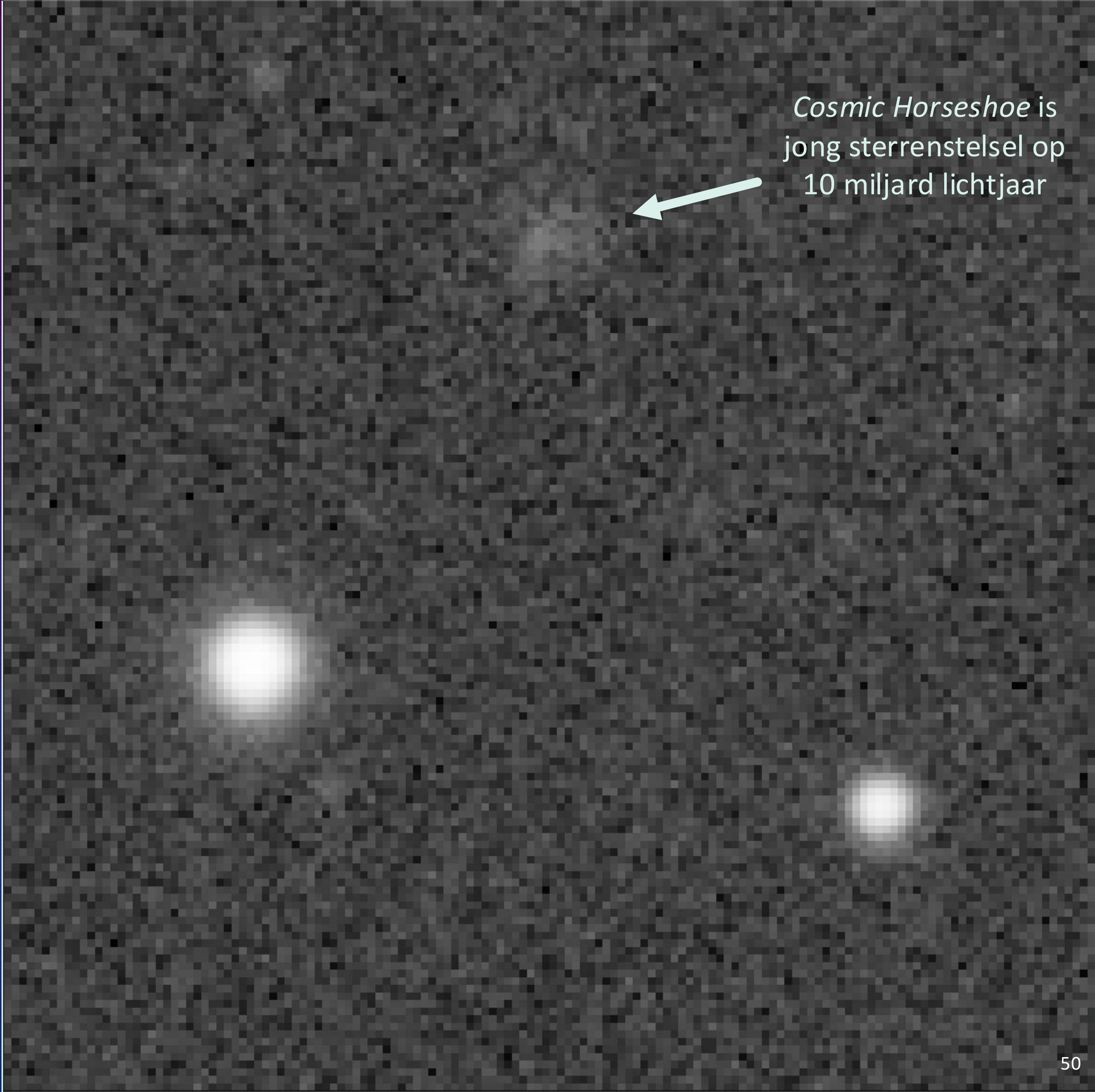
Cosmic Horseshoe:

- 38 Elektronen per pixel boven de ruis.
- Circa 10” diameter.

Circa 8500 fotonen van 10 miljard lichtjaar ver!

CONCLUSIES

- Cosmic Horseshoe lijkt zichtbaar met een kleine telescoop.
- Detectie van 10 miljard jaar verre fotonen.
- Betere opnames zijn een uitdaging!



Cosmic Horseshoe is jong sterrenstelsel op 10 miljard lichtjaar

REFERENTIES

- Met dank aan:
    - Robbert-Jan Westerduin
    - Frits van Loenen
    - Vereniging *Saturnus* en *Werkgroep AstroFotografie*
    - Christiaan Brinkerink (*Radboud Universiteit*)

- B. Eggen, D. Gomon, N. Sabti; Observing the Cosmic Horseshoe; Observing the Cosmic Horseshoe – Observing proposal - Group II, Leiden University, 2015-03-25.
- S. Schuldt et al.; Inner dark matter distribution of the Cosmic Horseshoe (J1148+1930) with gravitational lensing and dynamics; Astronomy & Astrophysics, A40, 2019.
- Johan van Dorp; S/N-ratio in deep-sky-opnamen - analyse en optimalisatie, ASTRO Bulletin, nr. 61, oktober 2019.

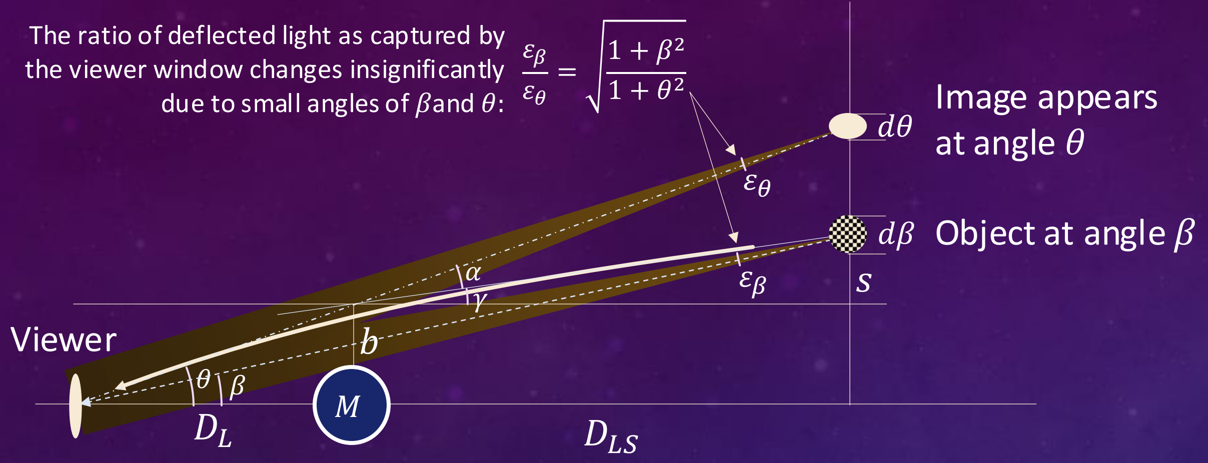
The End, so far.

APPENDIX

Light deflection in bended space-time

(circular symmetric lensing with point mass)

The ratio of deflected light as captured by the viewer window changes insignificantly due to small angles of β and θ: $\frac{\varepsilon_\beta}{\varepsilon_\theta} = \sqrt{\frac{1+\beta^2}{1+\theta^2}}$



An object at angle β, appears at angle θ as the mass M's gravity deflects the light by an angle α: $\alpha = \frac{4G}{c^2}\frac{M}{b}$

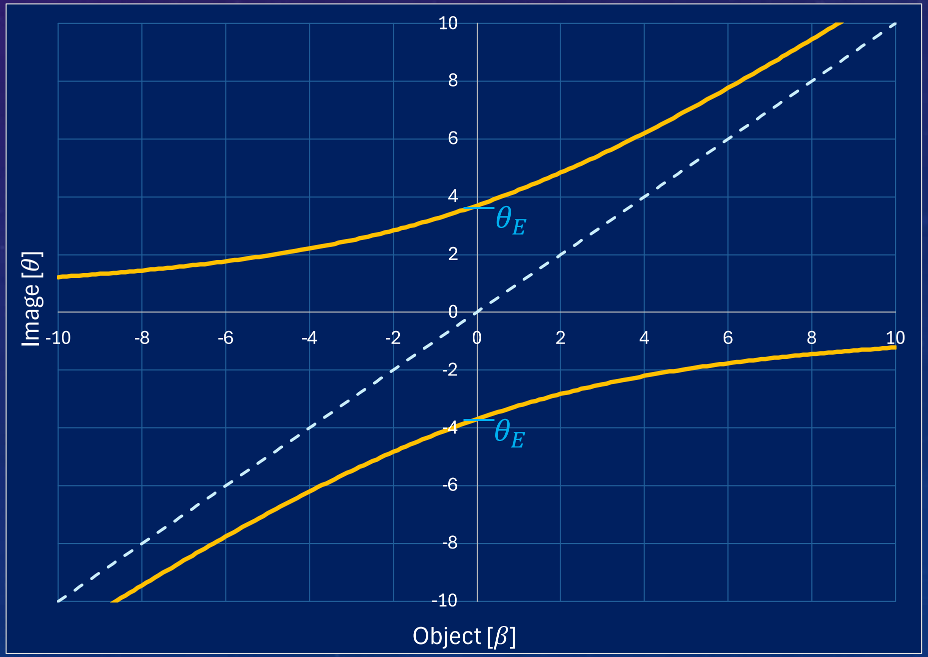
For small angles:

$$b = \theta \cdot D_L \qquad s = \gamma \cdot D_{LS} \qquad \gamma = \theta - \alpha$$

$$\beta \cdot (D_L + D_{LS}) = b + s = \theta \cdot D_L + \theta \cdot D_{LS} - \alpha \cdot D_{LS}$$

$$\Rightarrow \beta = \theta - \alpha \frac{D_{LS}}{D_L + D_{LS}} = \theta - \frac{4G}{c^2}\frac{M}{D_L} \cdot \frac{D_{LS}}{D_L + D_{LS}} \cdot \frac{1}{\theta}$$

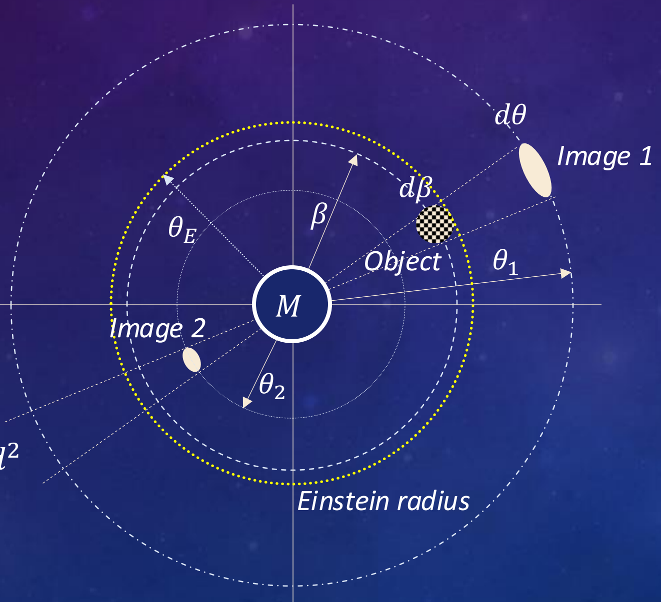
$$\boxed{\beta = \theta - \frac{\theta_E^2}{\theta}} \qquad \Rightarrow \theta_{1,2} = \frac{\beta \pm \sqrt{\beta^2 + 4\theta_E^2}}{2}$$



With a point mass, for every object at angle β there are two images at angles θ_1 and θ_2.

With $\beta = 0$ there is one image as a ring at the *Einstein radius* of 8.8E20 m or $\theta_E = 3.7''$ for the Cosmic Horseshoe:

$$\theta_E^2 = \frac{4G}{c^2}\frac{M}{D_L} \cdot \frac{D_{LS}}{D_L + D_{LS}} = 3.2 \cdot 10^{-10}\ rad^2$$

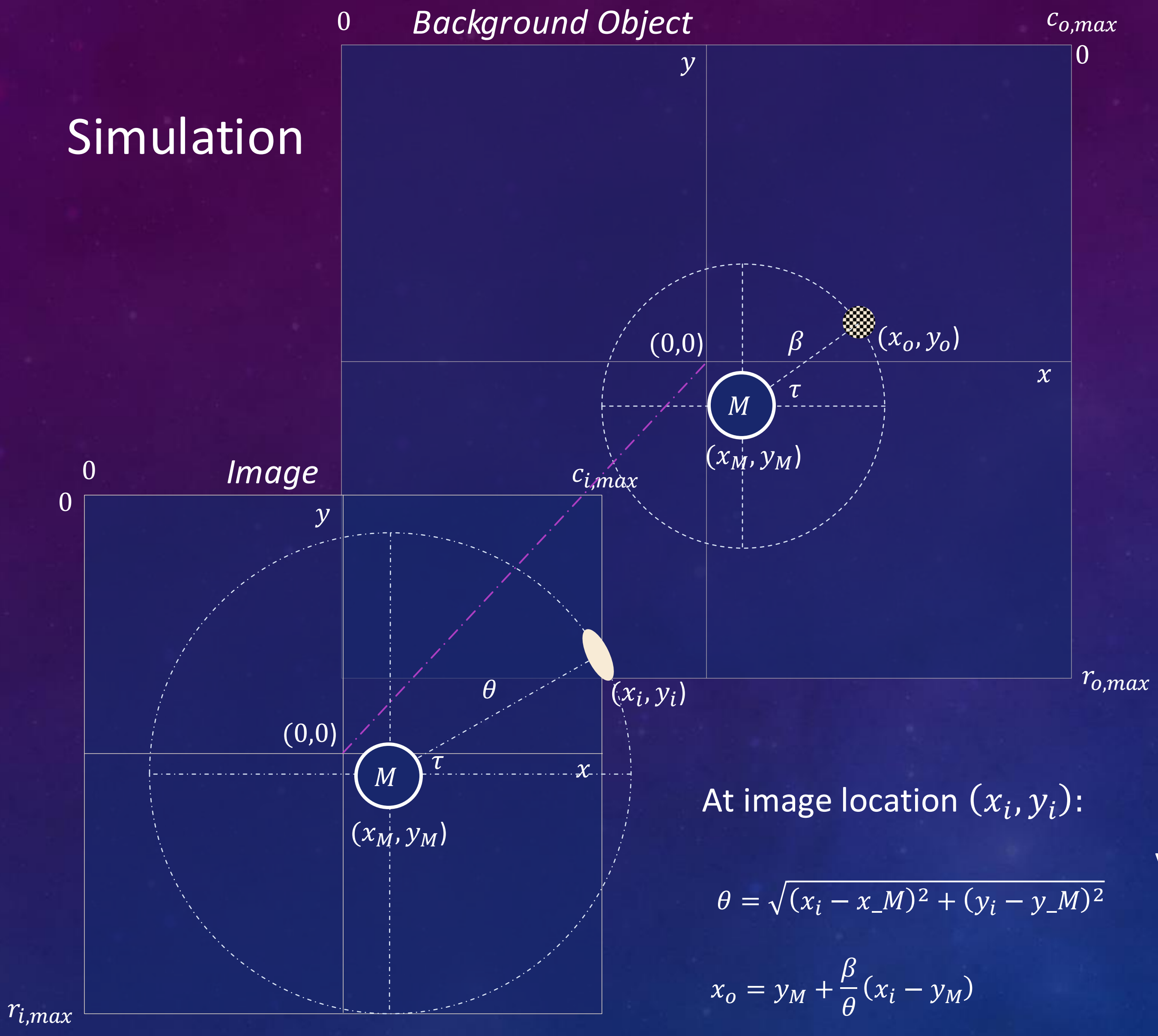


Field of view

Not to confuse with the Schwarzschild radius where the escape velocity equals the speed of light:

$$r_s = \frac{2GM}{c^2} = 1.6E16\ m\ or\ 0.000067''$$

Simulation



Construct image in Python simulation

1. Scan *Image* pixels from row $r_i = 0$ to $r_{i,max}$ and from column $c_i = 0$ to $c_{i,max}$ according to Python image coordinates.
2. Calculate from *Image* location (x_i, y_i) - scaled from pixel location (r_i, c_i) - the deflected angular distance θ to mass M (x_M, y_M).
3. Derive the angular angle β at the object from the deflected angular distance θ using equation $\beta = \theta\text{-}\frac{\theta_E^2}{\theta}$ and calculate object location (x_o, y_o).
4. Scale the *Background Object* location (x_o, y_o) to *Background Object* pixel location (r_o, c_o)
5. Set pixel brightness of *Image*(r_i, c_i) to *Background Object*(r_o, c_o).

At image location (x_i, y_i):

$$\theta = \sqrt{(x_i - x\_M)^2 + (y_i - y\_M)^2}$$

$$x_o = y_M + \frac{\beta}{\theta}(x_i - y_M)$$

$$y_o = y_M + \frac{\beta}{\theta}(y_i - y_M)$$

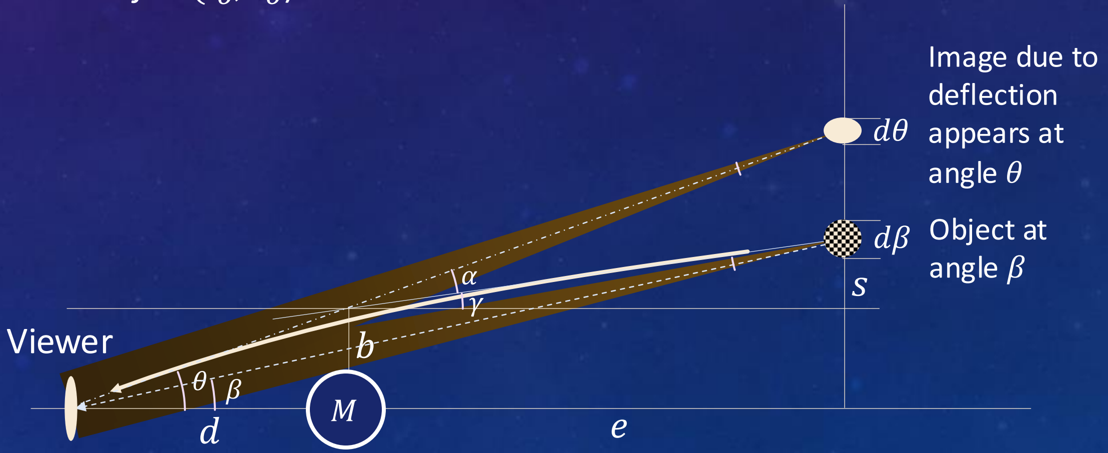


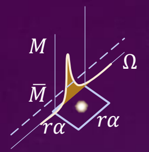
Image due to deflection appears at angle θ

Object at angle β

$$\boxed{\beta = \theta\text{-}\frac{\theta_E^2}{\theta}}$$

$$\theta_E^2 = \frac{4G}{c^2}\frac{M}{D_L} \cdot \frac{D_{LS}}{D_L + D_{LS}}$$

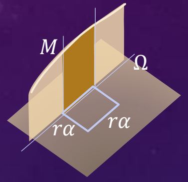
Star brightness: magnitude

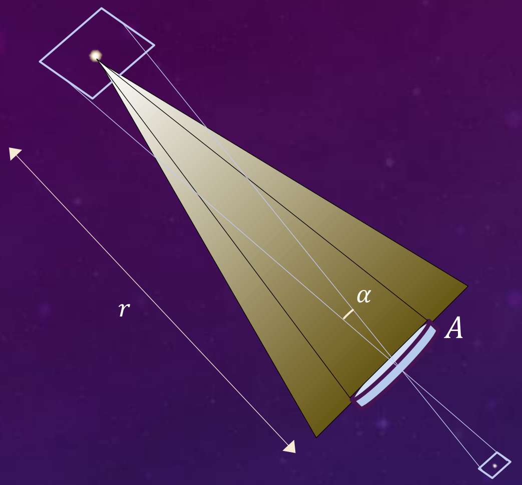


Definitions

Φ Luminous flux, Lumen, Luminous energy per unit time
M Luminous exitance, Lumen/m^2, Luminous flux *emitted* from a surface
E Illuminance, Lux = Lumen/m^2, Luminous flux *incident* on a surface
For Vega ($m_V = 0$) illuminance = 1.6E-6 Lux, $E_V = 3.2 \cdot 10^{10}$ photons/m^2/s

Nebula brightness: magnitude per arcsecond^2



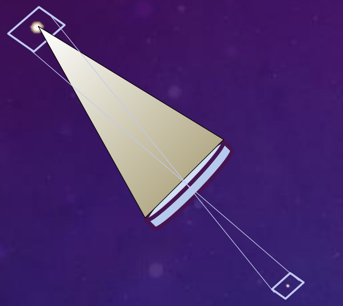


The amount of starlight captured by a sensor is independent of the solid pixel angle α.

A star is like a point source with an overall luminous flux Φ.
The illuminance at the sensor depends on the star distance r only: $E_s = \frac{\Phi_s}{4\pi r^2}$

The fraction of the luminous flux from a star captured by the telescope lens A is:

$$\Phi_A = \frac{\Phi_s}{4\pi r^2} \cdot \frac{\pi A^2}{4}$$

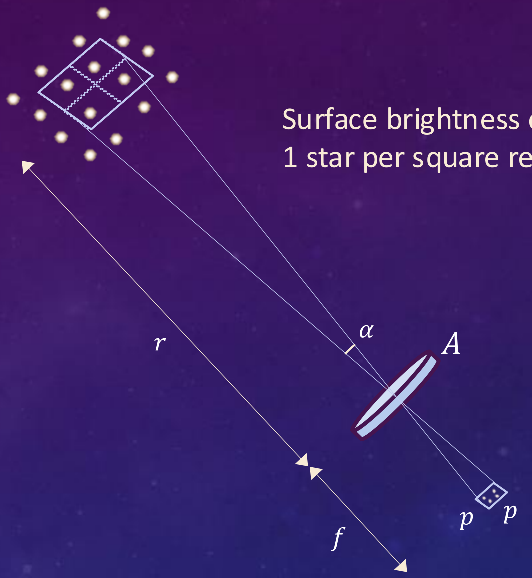


A closer star illuminates brighter at the telescope.

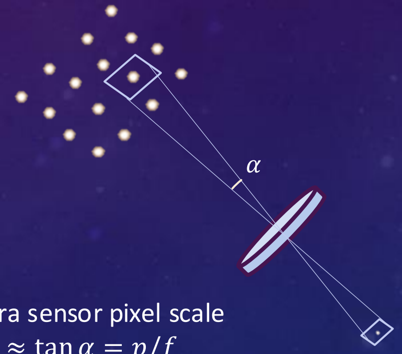
The illuminance from a star of magnitude m by the telescope lens area A is measured relative to the illuminance of Vega: $E_V = \frac{\Phi_V}{4\pi r^2}$

The telescope captures a luminous flux:

$$\Phi_A = E_V \cdot 100^{-(m-m_V)/5} \cdot \frac{\pi A^2}{4}$$



Surface brightness equivalent of 1 star per square remains.



Camera sensor pixel scale
$\alpha \approx \tan\alpha = p/f$

More light from a nebula is captured by a sensor with a larger solid pixel angle α.

Nebula luminous exitance $M_N = {d^2\Phi_N}/{d(r\,\alpha)^2}$ is emitted flux per *square meter* of nebula.
The luminous flux Φ_n of a nebula:

$$\Phi_N = \iint_0^{\alpha^2} M(\Omega) r^2 d\Omega^2 = \bar{M}_N r^2 \alpha^2$$

(called the etendue).

With a shorter focal length, the pixel angle α becomes larger and consequently the luminous flux Φ_n becomes larger.

With a nebula closer, a smaller area of the nebula is observed, and the surface brightness is preserved.

The illuminance at the sensor is independent of the distance of the nebula:

$$E_N = \frac{\Phi_N}{4\pi r^2} = \frac{\bar{M}_N \alpha^2}{4\pi}$$

The telescope captures a luminous flux:

$$\Phi_A = (E_V/\alpha_V^2) \cdot 100^{-(m_N-m_V)/5} \cdot \alpha^2 \cdot \frac{\pi A^2}{4}$$

Light intensity through a gravitation lens

(circular symmetric lensing with point mass)

For resolved objects ('nebula') the observed surface brightness does not change with magnification. This is because the magnification changes the observed area of the object, but the object solid angle of the observer will counterbalance this effect, and the observed surface brightness remains.

For for unresolved objects (stars) the magnification due to lensing changes the observed brightness. The magnification of the gravitation lens μ is the ratio of the image to the object area.

$$\mu = \frac{\theta \cdot \tau}{\beta \cdot \tau} \cdot \frac{d\theta}{d\beta}$$

With the relation of the object angle δ and image angle β:

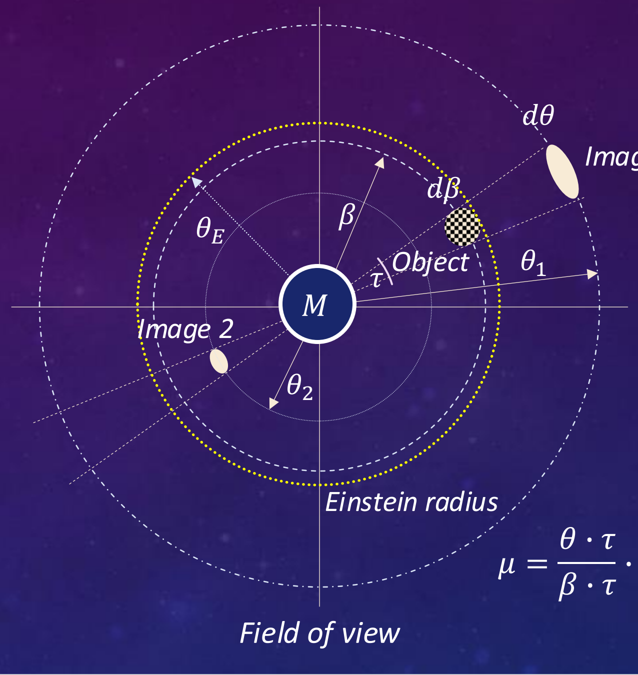
$$\beta = \theta - \frac{\theta_E^2}{\theta} \quad \text{or} \quad \theta_{1,2} = \frac{\beta \pm \sqrt{\beta^2 + 4\theta_E^2}}{2}$$

The magnification due to lensing is (with $u = \beta / \theta_E$)[1]:

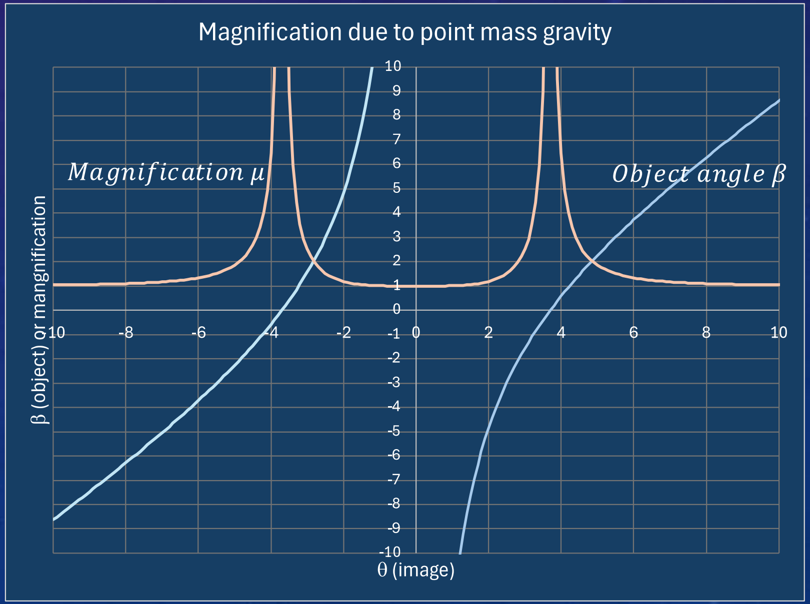
$$\mu_{1,2} = \left(\left(1 - \frac{\theta_E^2}{{\theta_{1,2}}^2}\right)\left(1 + \frac{\theta_E^2}{{\theta_{1,2}}^2}\right)\right)^{-1} = \left(1 - \left(\frac{\theta_E}{\theta_{1,2}}\right)^4\right)^{-1} = \frac{u^2+2}{2u\sqrt{u^2+4}} \pm \frac{1}{2}$$

The total observed magnification is, with both images, $\mu_{tot} = |\mu_1| + |\mu_2|$

$$\mu_{tot} = \frac{u^2+2}{u\sqrt{u^2+4}}$$



$$\mu = \frac{\theta \cdot \tau}{\beta \cdot \tau} \cdot \frac{d\theta}{d\beta}$$



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[1] https://arxiv.org/abs/astro-ph/9812021 with equations (17) and (19)

HORSESHOE AMATEUR OBSERVATIONS (LARGE TELESCOPES)

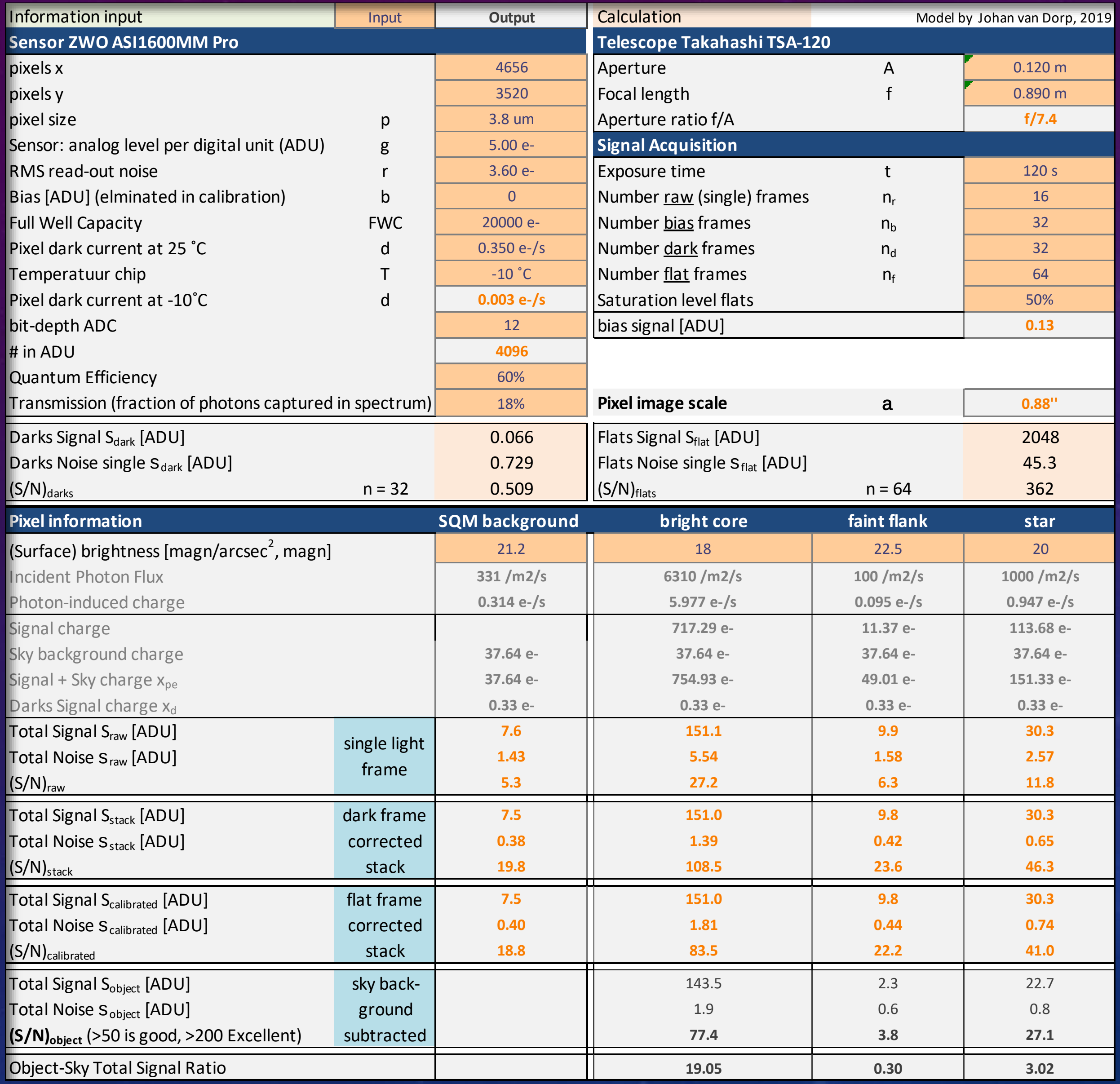
Bart Leclerq, 4 h, 500 mm Newton

Jerry Macon, 31h, 350 mm Ritchey-Cretien

Morten la Cour, 18 h, 300 mm Newton

Marc Dickinson, 26 h, 350 mm Cassegrain

SIGNAAL / RUIS OPTIMALISATIE



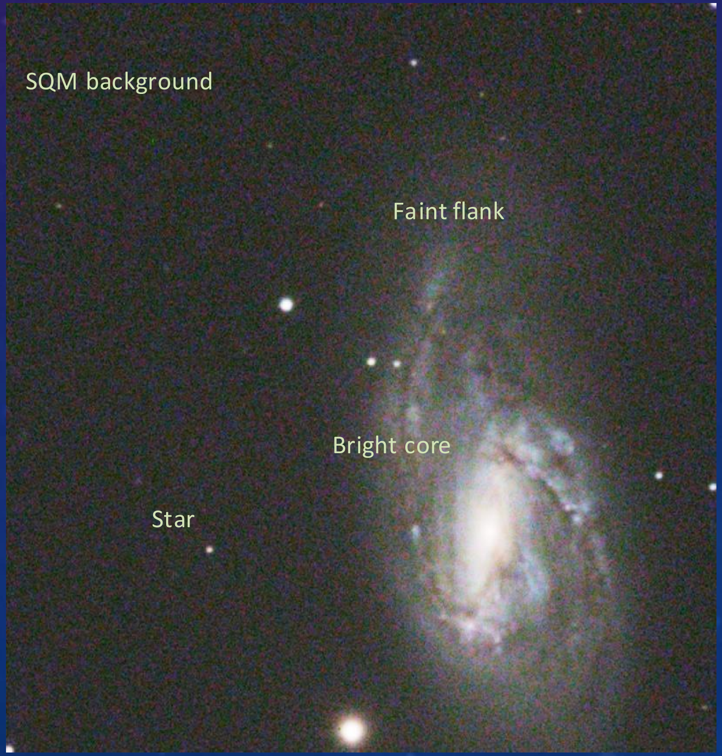
Information input	Input	Output	Calculation		Model by Johan van Dorp, 2019
Sensor ZWO ASI1600MM Pro			**Telescope Takahashi TSA-120**		
pixels x		4656	Aperture	A	0.120 m
pixels y		3520	Focal length	f	0.890 m
pixel size	p	3.8 um	Aperture ratio f/A		f/7.4
Sensor: analog level per digital unit (ADU)	g	5.00 e-	**Signal Acquisition**		
RMS read-out noise	r	3.60 e-	Exposure time	t	120 s
Bias [ADU] (elminated in calibration)	b	0	Number raw (single) frames	n_r	16
Full Well Capacity	FWC	20000 e-	Number bias frames	n_b	32
Pixel dark current at 25 ˚C	d	0.350 e-/s	Number dark frames	n_d	32
Temperatuur chip	T	-10 ˚C	Number flat frames	n_f	64
Pixel dark current at -10˚C	d	0.003 e-/s	Saturation level flats		50%
bit-depth ADC		12	bias signal [ADU]		0.13
# in ADU		4096			
Quantum Efficiency		60%			
Transmission (fraction of photons captured in spectrum)		18%	**Pixel image scale**	a	0.88''
Darks Signal S_{dark} [ADU]		0.066	Flats Signal S_{flat} [ADU]		2048
Darks Noise single s_{dark} [ADU]		0.729	Flats Noise single s_{flat} [ADU]		45.3
$(S/N)_{darks}$	n = 32	0.509	$(S/N)_{flats}$	n = 64	362

Pixel information		SQM background	bright core	faint flank	star
(Surface) brightness [magn/arcsec^2, magn]		21.2	18	22.5	20
Incident Photon Flux		331 /m2/s	6310 /m2/s	100 /m2/s	1000 /m2/s
Photon-induced charge		0.314 e-/s	5.977 e-/s	0.095 e-/s	0.947 e-/s
Signal charge			717.29 e-	11.37 e-	113.68 e-
Sky background charge		37.64 e-	37.64 e-	37.64 e-	37.64 e-
Signal + Sky charge x_{pe}		37.64 e-	754.93 e-	49.01 e-	151.33 e-
Darks Signal charge x_d		0.33 e-	0.33 e-	0.33 e-	0.33 e-
Total Signal S_{raw} [ADU]	single light frame	7.6	151.1	9.9	30.3
Total Noise s_{raw} [ADU]		1.43	5.54	1.58	2.57
$(S/N)_{raw}$		5.3	27.2	6.3	11.8
Total Signal S_{stack} [ADU]	dark frame corrected stack	7.5	151.0	9.8	30.3
Total Noise s_{stack} [ADU]		0.38	1.39	0.42	0.65
$(S/N)_{stack}$		19.8	108.5	23.6	46.3
Total Signal $S_{calibrated}$ [ADU]	flat frame corrected stack	7.5	151.0	9.8	30.3
Total Noise $s_{calibrated}$ [ADU]		0.40	1.81	0.44	0.74
$(S/N)_{calibrated}$		18.8	83.5	22.2	41.0
Total Signal S_{object} [ADU]	sky back-ground subtracted		143.5	2.3	22.7
Total Noise s_{object} [ADU]			1.9	0.6	0.8
$(S/N)_{object}$ (>50 is good, >200 Excellent)			77.4	3.8	27.1
Object-Sky Total Signal Ratio			19.05	0.30	3.02

Model

Ontwikkeld door Johan van Dorp (ASTROBulletin 61, 2019).

Berekent de signaal-ruis verhouding voor een telescoop met camera.



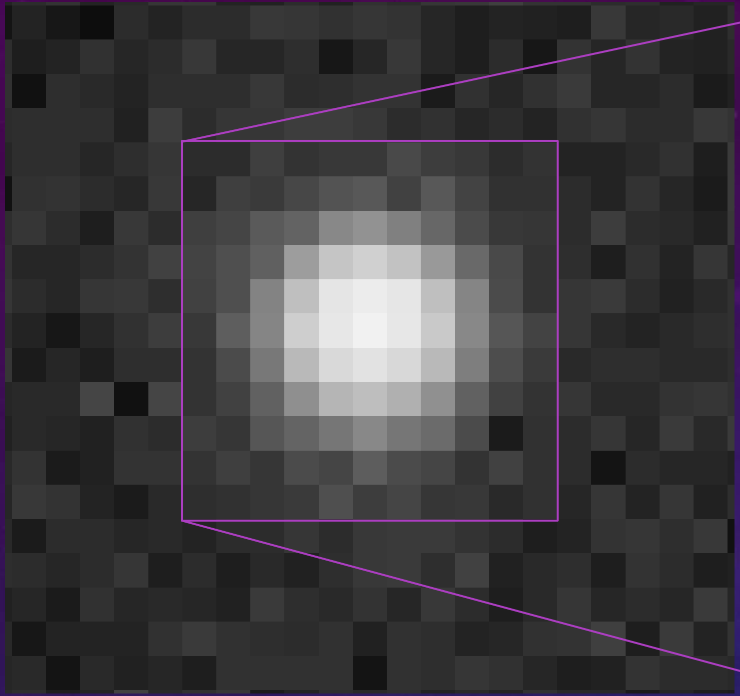
Star light center of gravity

The spreading of the star light, the Point Spread Function (PSF), is defined by optical diffraction, atmospheric diffraction and photon Poisson noise, and is close to a Gaussian distribution function.

Independent of focus, with the momentum equation the center of the star can be calculated accurately (5.993, 6.104) with star light P_i above the background average S:

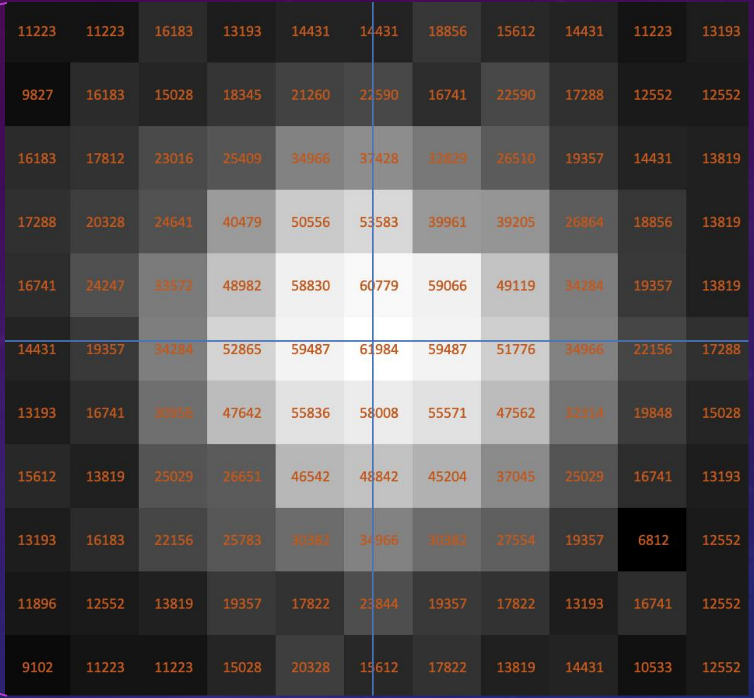
$$\bar{x} = \frac{\sum_i x_i (P_i - S)}{\sum_i (P_i - S)} \qquad \bar{y} = \frac{\sum_i y_i (P_i - S)}{\sum_i (P_i - S)}$$

With light per pixel measured, the center of a star can be measured with more precision than the resolution of the camera or telescope.



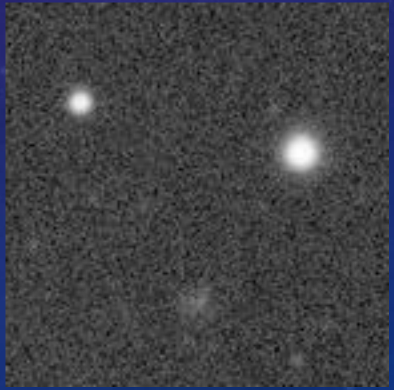
Star light on a single 120 s exposed luminance frame with 120 mm refractor in frame center.

1 pixel is 3.8 µm or 0.89”

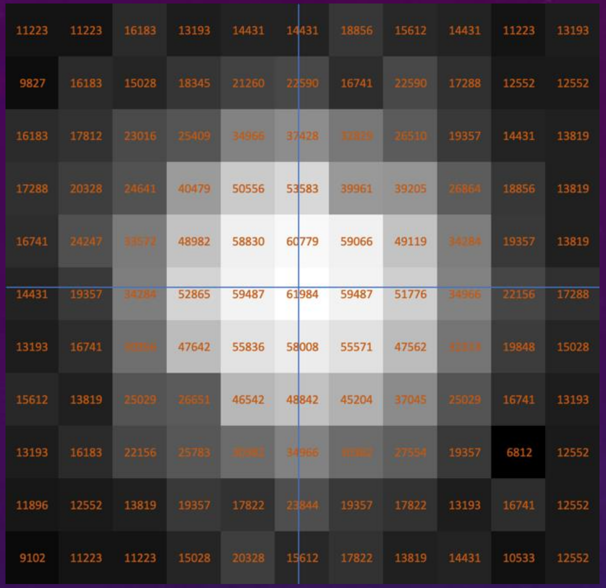


11223	11223	16183	13193	14431	14431	18856	15612	14431	11223	13193
9827	16183	15028	18345	21260	22590	16741	22590	17288	12552	12552
16183	17812	23016	25409	34966	37428	32829	26510	19357	14431	13819
17288	20328	24641	40479	50556	53583	39961	39205	26864	18856	13819
16741	24247	33572	48982	58830	60779	59066	49119	34284	19357	13819
14431	19357	34284	52865	59487	61984	59487	51776	34966	22156	17288
13193	16741	30956	47642	55836	58008	55571	47562	32314	19848	15028
15612	13819	25029	26651	46542	48842	45204	37045	25029	16741	13193
13193	16183	22156	25783	30382	34966	30382	27554	19357	6812	12552
11896	12552	13819	19357	17822	23844	19357	17822	13193	16741	12552
9102	11223	11223	15028	20328	15612	17822	13819	14431	10533	12552

Star light values (ADU) scaled to 16 bit.

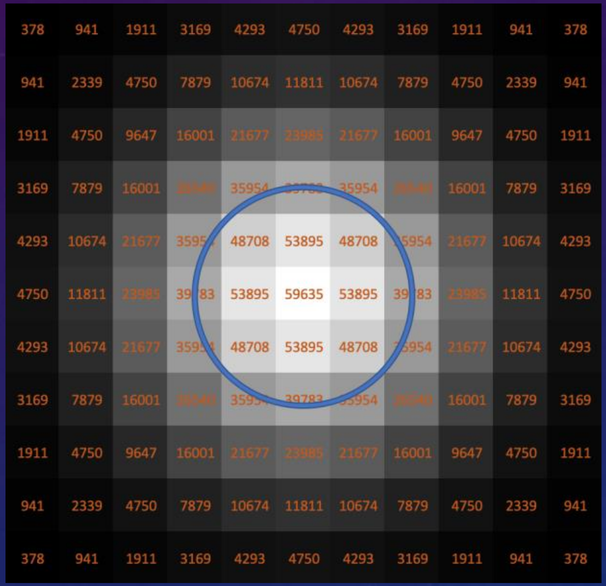


Star light FWHM model (former)



11223	11223	16183	13193	14431	14431	18856	15612	14431	11223	13193
9827	16183	15028	18345	21260	22590	16741	22590	17288	12552	12552
16183	17812	23016	25409	34966	37428	[illegible]	26510	19357	14431	13819
17288	20328	24641	40479	50556	53583	39961	39205	26864	18856	13819
16741	24247	33572	48982	58830	60779	59066	49119	34284	19357	13819
14431	19357	34284	52865	59487	61984	59487	51776	34966	22156	17288
13193	16741	[illegible]	47642	55836	58008	55571	47562	[illegible]	19848	15028
15612	13819	25029	26651	46542	48842	45204	37045	25029	16741	13193
13193	16183	22156	25783	[illegible]	34966	[illegible]	27554	19357	6812	12552
11896	12552	13819	19357	17822	23844	19357	17822	13193	16741	12552
9102	11223	11223	15028	20328	15612	17822	13819	14431	10533	12552

Star light values (ADU) scaled to 16 bit.



378	941	1911	3169	4293	4750	4293	3169	1911	941	378
941	2339	4750	7879	10674	11811	10674	7879	4750	2339	941
1911	4750	9647	16001	21677	23985	21677	16001	9647	4750	1911
3169	7879	16001	28540	35954	39783	35954	28540	16001	7879	3169
4293	10674	21677	3595[illegible]	48708	53895	48708	[illegible]5954	21677	10674	4293
4750	11811	23985	39[illegible]83	53895	59635	53895	39[illegible]83	23985	11811	4750
4293	10674	21677	3595[illegible]	48708	53895	48708	[illegible]5954	21677	10674	4293
3169	7879	16001	28540	3595[illegible]	39783	[illegible]5954	28540	16001	7879	3169
1911	4750	9647	16001	21677	23985	21677	16001	9647	4750	1911
941	2339	4750	7879	10674	11811	10674	7879	4750	2339	941
378	941	1911	3169	4293	4750	4293	3169	1911	941	378

Star light modelled in Gaussian distribution with FWHM shown as blue circle (3.5 pixels).

The spread of the star light, the Point Spread Function (PSF), is defined by optical diffraction, atmospherically diffraction and photon Poisson noise, and is close to a Gaussian distribution function.

The light P above the background average S is modelled empirically:

$$P_r = P_0 \cdot e^{-\left(\frac{r}{\sigma}\right)^2}$$

By rewriting the equation in a linear function, with curve fitting parameters P_0 and σ can be determined.

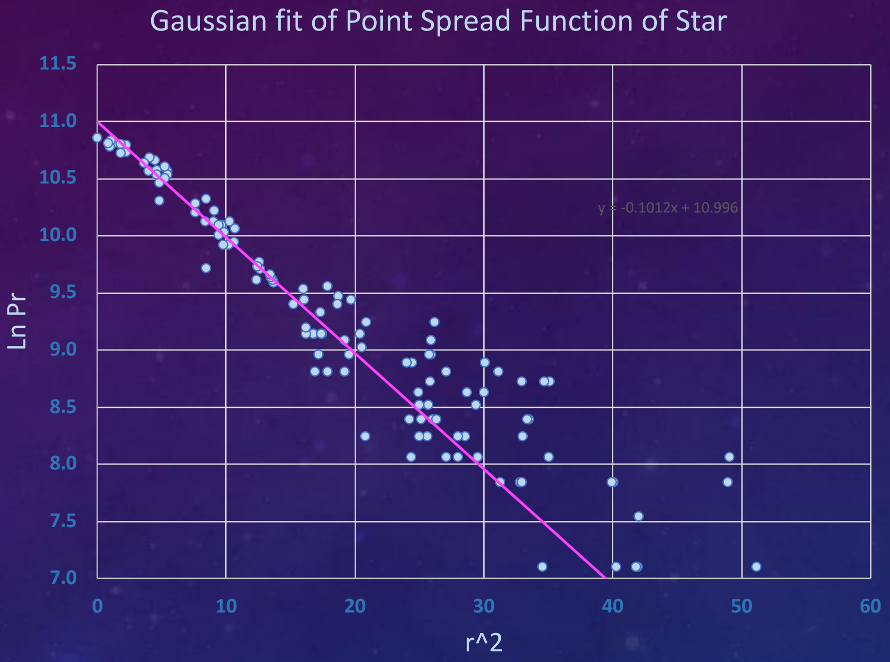
From the linear fitted function $y = ax + b$:

$$y = \ln P_r \quad x = r^2 \quad a = -\frac{1}{\sigma^2} \quad b = \ln P_0$$

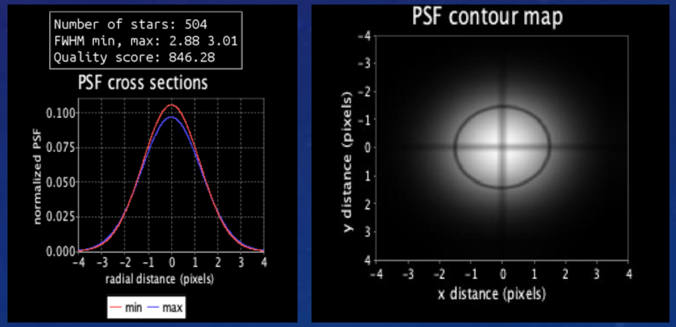
The PSF is characterized by the Full-Width at Half the Maximum (FWHM).

From the Gaussian distribution function, with $P_r = P_0/2$ the FWHM can easily be derived as:

$$FWHM = 2 \cdot \sqrt{Ln\,2} \cdot \sigma$$

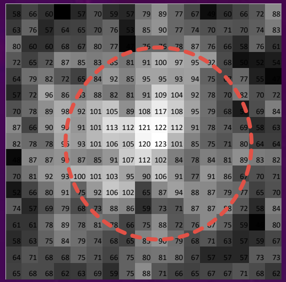


Fitting the Gaussian distribution on the star light curve based on light per pixel received. Small errors can be seen, demonstrating that the curve is not exactly Gaussian, e.g. by Airy-disc or guiding control.

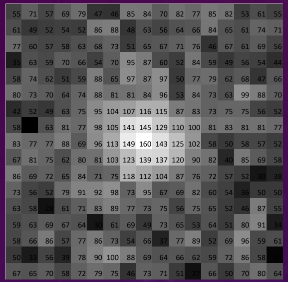


```
Number of stars: 504
FWHM min, max: 2.88 3.01
Quality score: 846.28
```

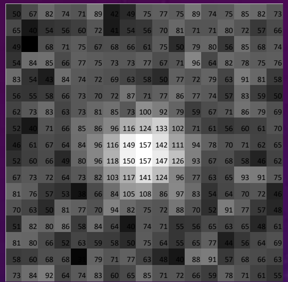
The image processing program APP generates a measured PSF distribution curve as well as a contour map in both dimensions of all stars of one image. This will affect the 'image quality score'.



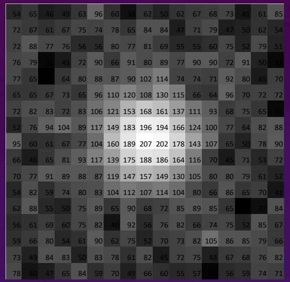
Einstein ring



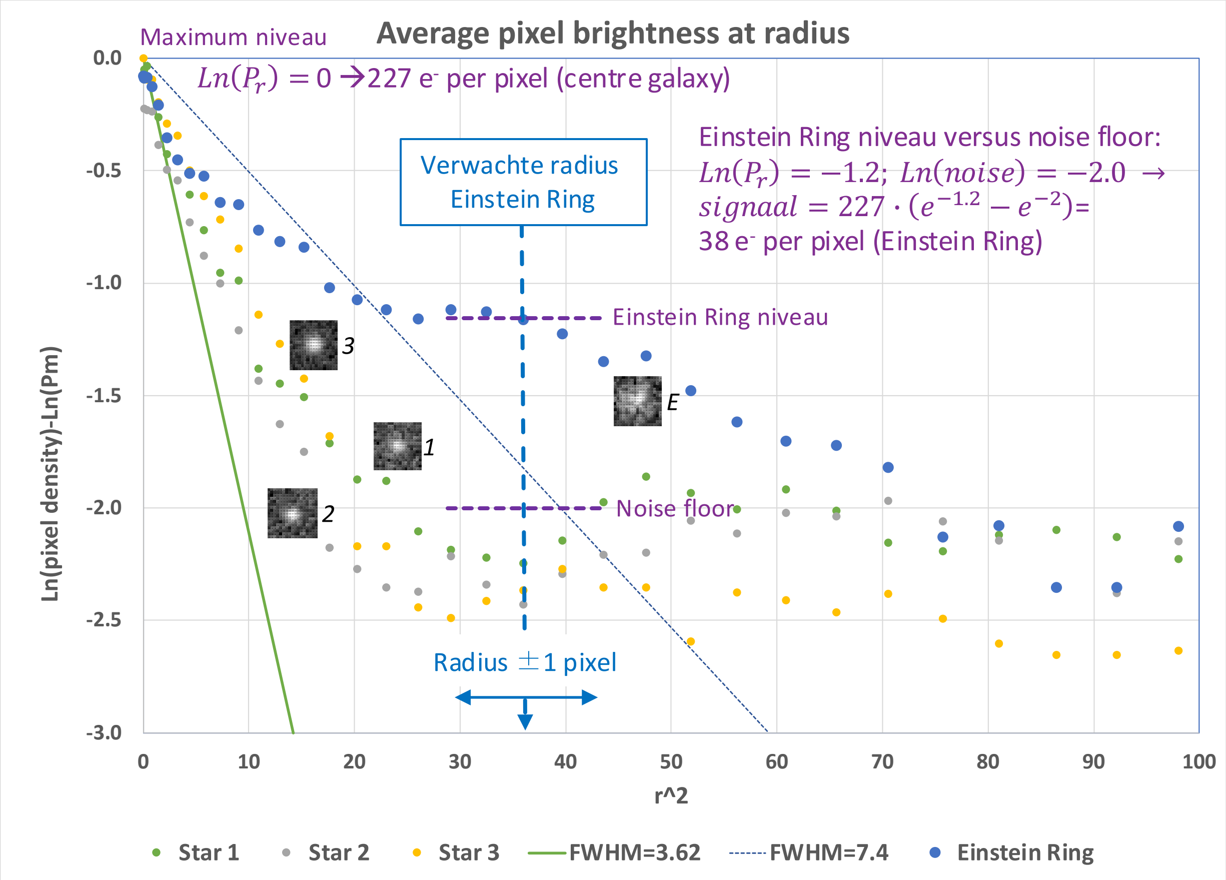
Reference Star 1

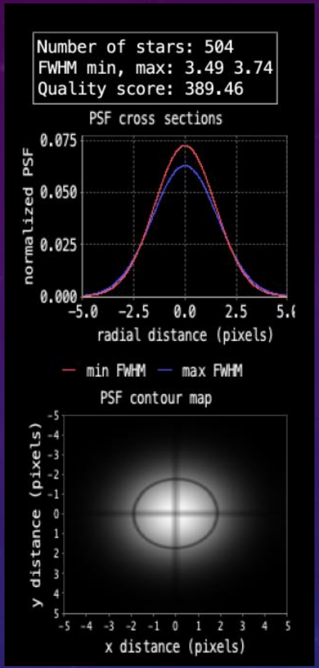


Reference Star 2



Reference Star 3





```
Number of stars: 504
FWHM min, max: 3.49 3.74
Quality score: 389.46
```

Star light spread

The spread of the star light, the Point Spread Function (PSF), is defined by optical diffraction, atmospherically diffraction and photon Poisson noise, and is close to a Gaussian distribution function.

$$P_r = P_m \cdot e^{-\left(\frac{r}{\sigma}\right)^2}$$

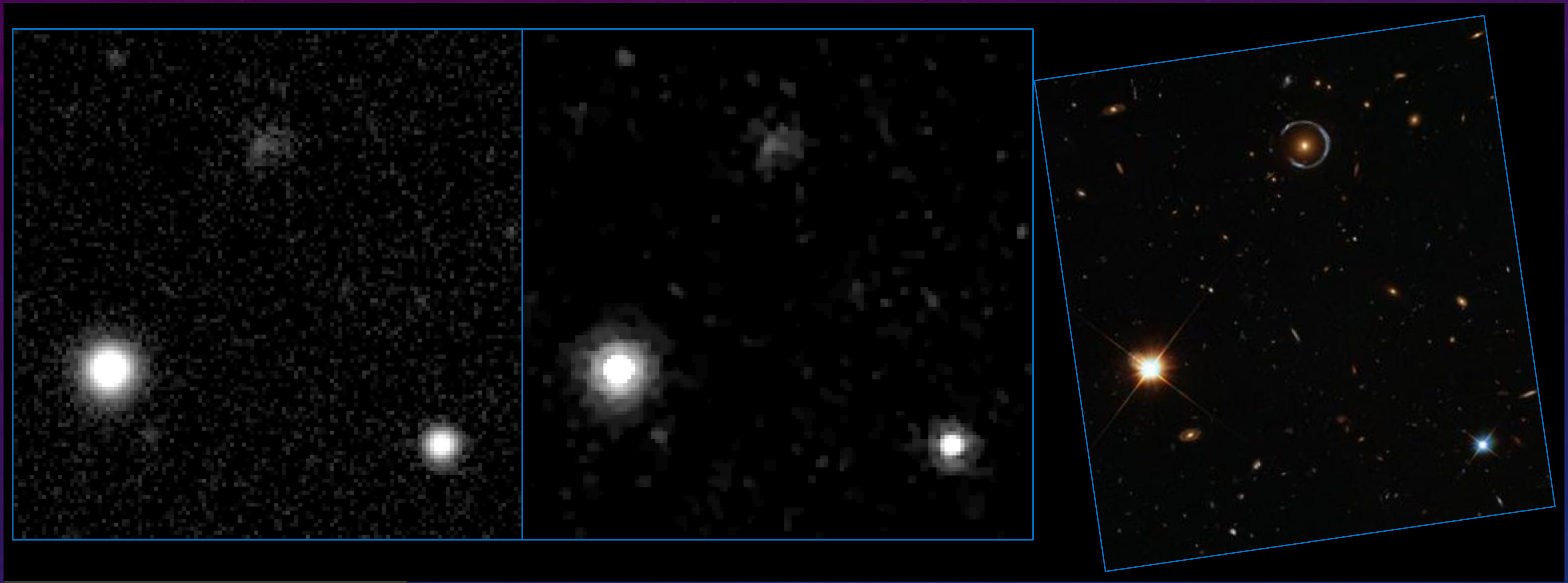
The PSF is characterized by the Full-Width at Half the Maximum (FWHM) and from this we can determine the spread σ of the Gaussian distribution function.

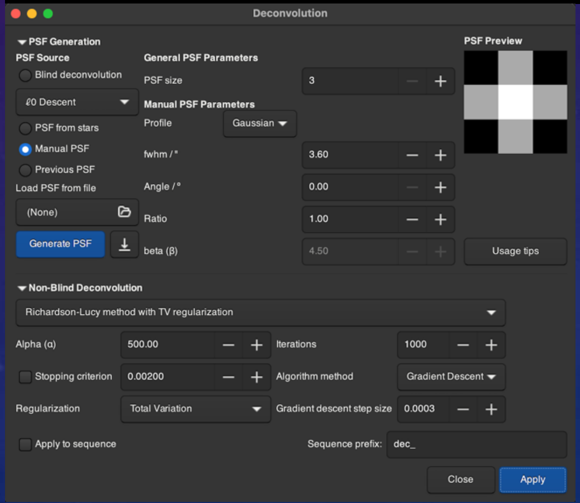
$$\sigma = \frac{FWHM}{2\sqrt{Ln\ 2}} \qquad a = -\frac{1}{\sigma^2} = \frac{-4 \cdot Ln\ 2}{FWHM^2}$$

With the average $FWHM = 3.62$ on average of the Einstein Ring photograph, $\sigma = 2.2$. and the slope is $a = -0.21$.

The galaxy at the front which form the lens appears to show a light fall off along the line FWHM=7.4 because it is not a point source.

About 10-20 pixels received light from the Einstein ring with at about 38 e^-/pixel. In total about 570 electrons. This corresponds to about 8500 photons captured (with 11% spectral transmission and quantum efficiency) at a rate of one photon every 3 s for eight hours long.





PSF Generation

PSF Source
○ Blind deconvolution
ℓ0 Descent
○ PSF from stars
● Manual PSF
○ Previous PSF
Load PSF from file: (None)

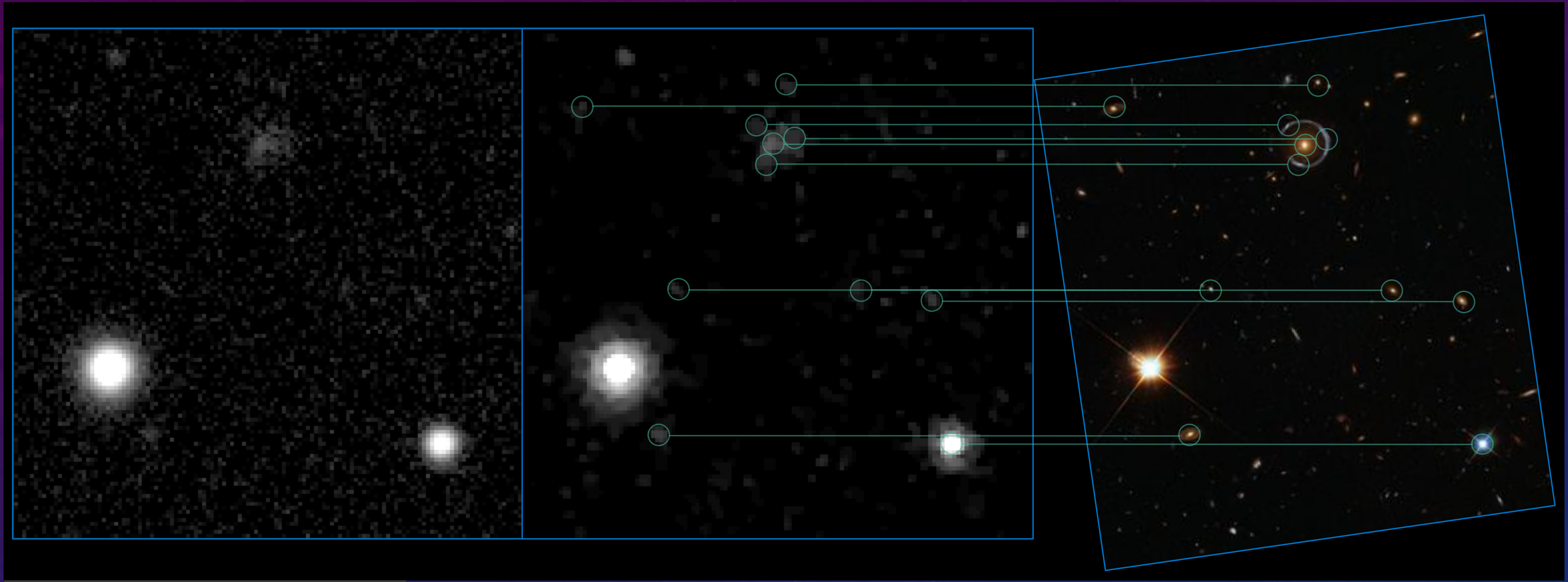
General PSF Parameters
PSF size: 3

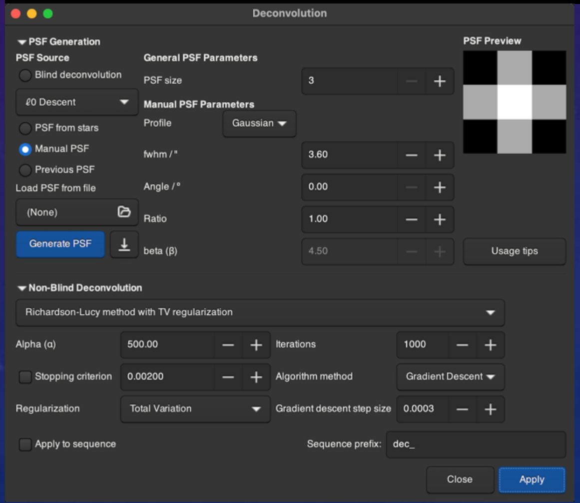
Manual PSF Parameters
Profile: Gaussian
fwhm / ": 3.60
Angle / °: 0.00
Ratio: 1.00
beta (β): 4.50

Non-Blind Deconvolution
Richardson-Lucy method with TV regularization
Alpha (α): 500.00
Iterations: 1000
☐ Stopping criterion: 0.00200
Algorithm method: Gradient Descent
Regularization: Total Variation
Gradient descent step size: 0.0003
☐ Apply to sequence
Sequence prefix: dec_

De Einstein Ring, links origineel met 120 mm kijker en 8 uur belicht, midden door deconvolutie in Siril en rechts opname met Hubble Space Telescope. De verbetering door deconvolutie is niet groot, maar de ring laat helderheidsvariaties zien waar de HST die ook ziet. Ook lichten pixels op bij sterrenstelsels van maar enkele boogseconden groot (1 pixel = 0.88”).

← Siril deconvolutie instellingen: PSF gemeten van ster op 3.6 pixels FWHM, 1000 iteraties met Richardson-Lucy methode.





Deconvolution

PSF Generation

PSF Source
○ Blind deconvolution
ℓ0 Descent
○ PSF from stars
● Manual PSF
○ Previous PSF
Load PSF from file: (None)

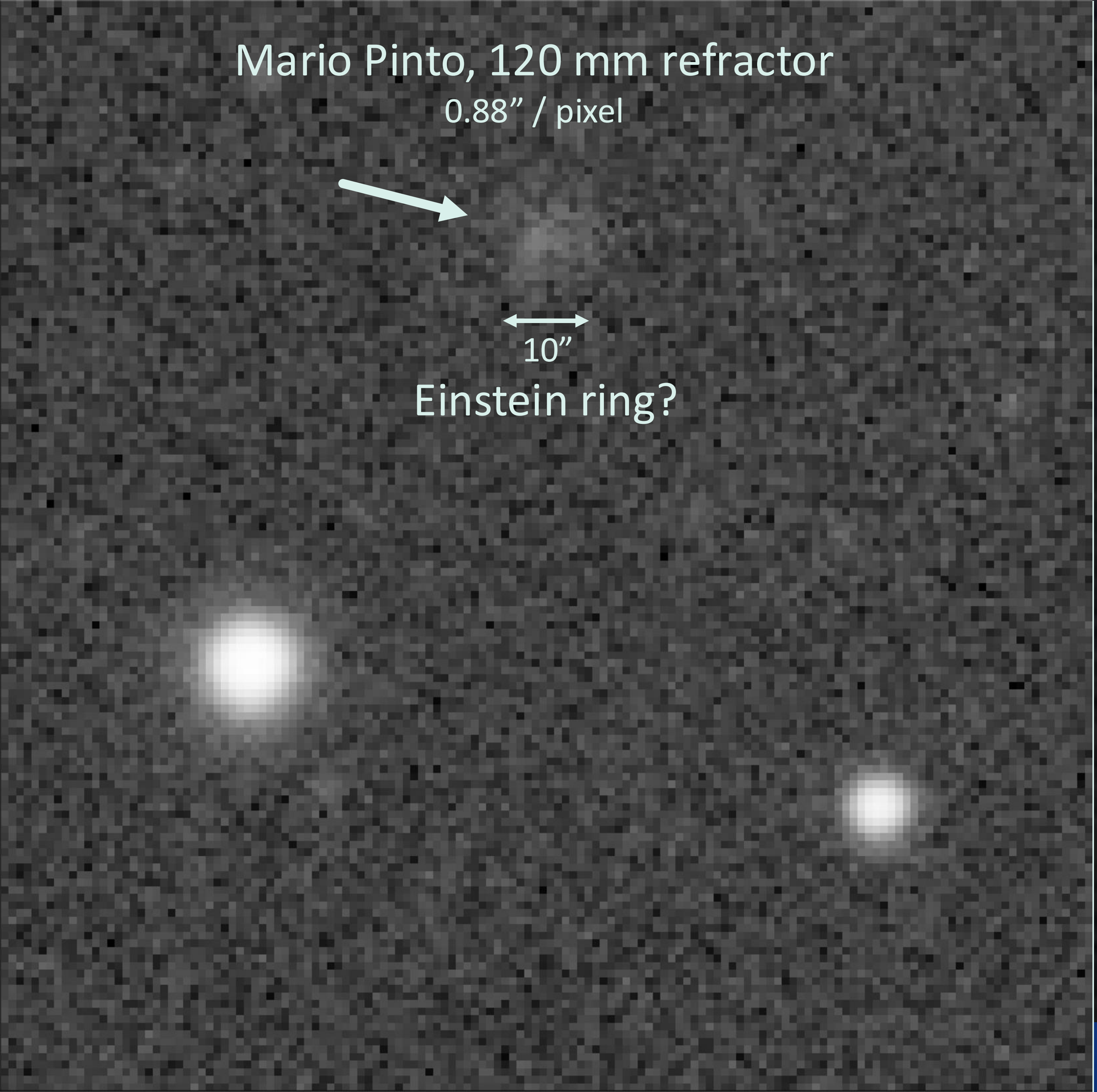
General PSF Parameters
PSF size: 3

Manual PSF Parameters
Profile: Gaussian
fwhm / ": 3.60
Angle / °: 0.00
Ratio: 1.00
beta (β): 4.50

Non-Blind Deconvolution
Richardson-Lucy method with TV regularization
Alpha (α): 500.00
Iterations: 1000
☐ Stopping criterion: 0.00200
Algorithm method: Gradient Descent
Regularization: Total Variation
Gradient descent step size: 0.0003
☐ Apply to sequence
Sequence prefix: dec_

De Einstein Ring, links origineel met 120 mm kijker en 8 uur belicht, midden door deconvolutie in Siril en rechts opname met Hubble Space Telescope. De verbetering door deconvolutie is niet groot, maar de ring laat helderheidsvariaties zien waar de HST die ook ziet. Ook lichten pixels op bij sterrenstelsels van maar enkele boogseconden groot (1 pixel = 0.88").

← Siril deconvolutie instellingen: PSF gemeten van ster op 3.6 pixels FWHM, 1000 iteraties met Richardson-Lucy methode.



Mario Pinto, 120 mm refractor

0.88″ / pixel

10″

Einstein ring?

Bart Leclercq, 500 mm f/4 Newton

48x5′, ZQO ASI 294MC Pro