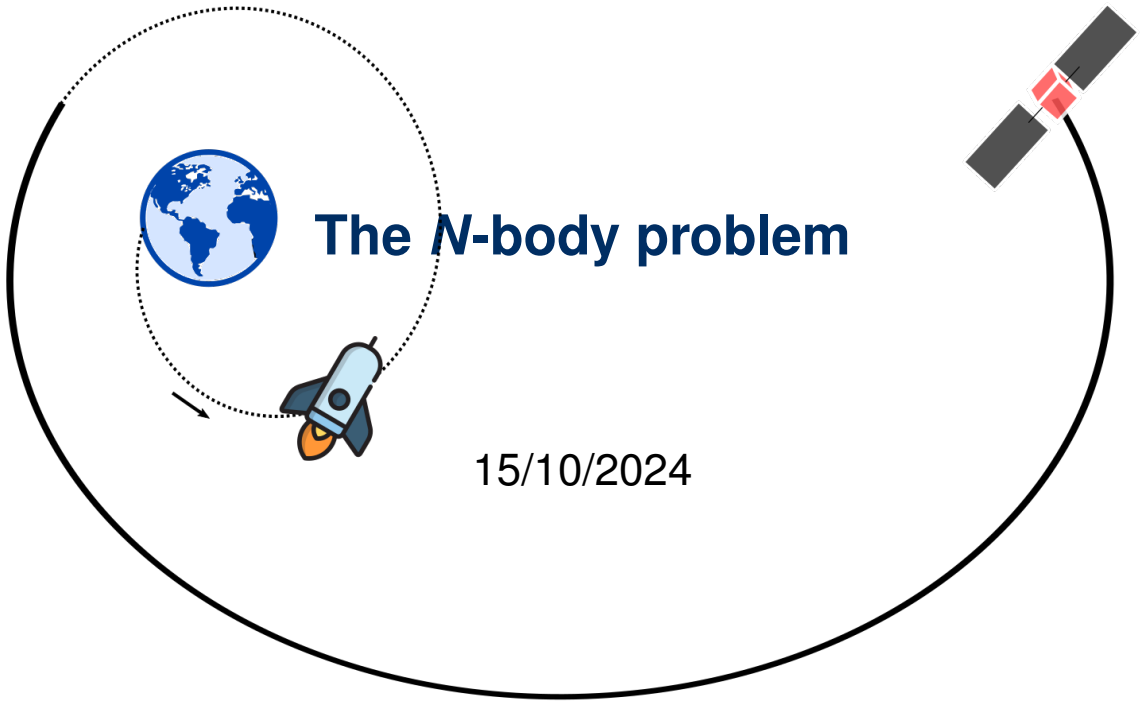
How do things move in Space?

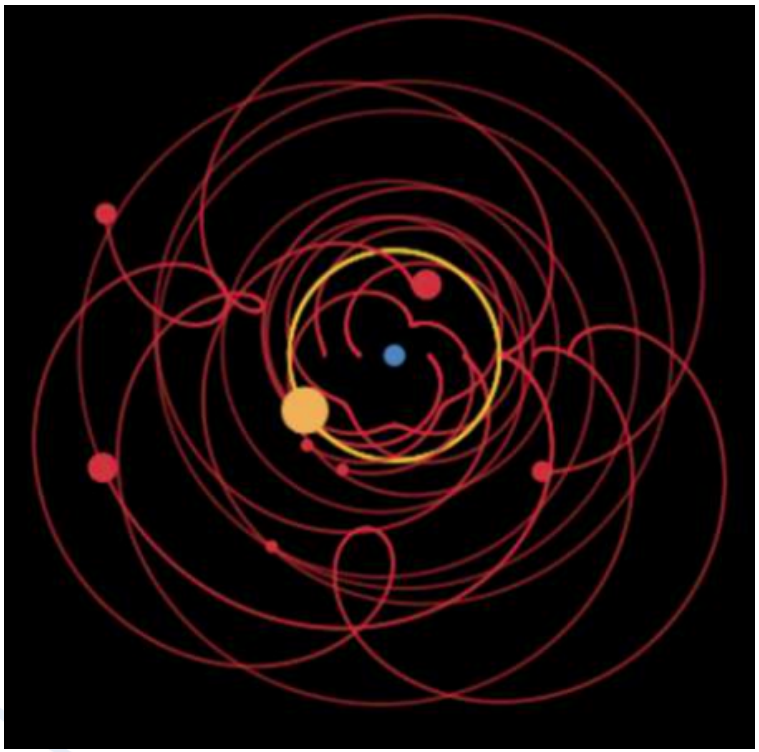
Verónica Saz Ulibarrena



15/10/2024

Historical introduction : Copernicus

Geocentric



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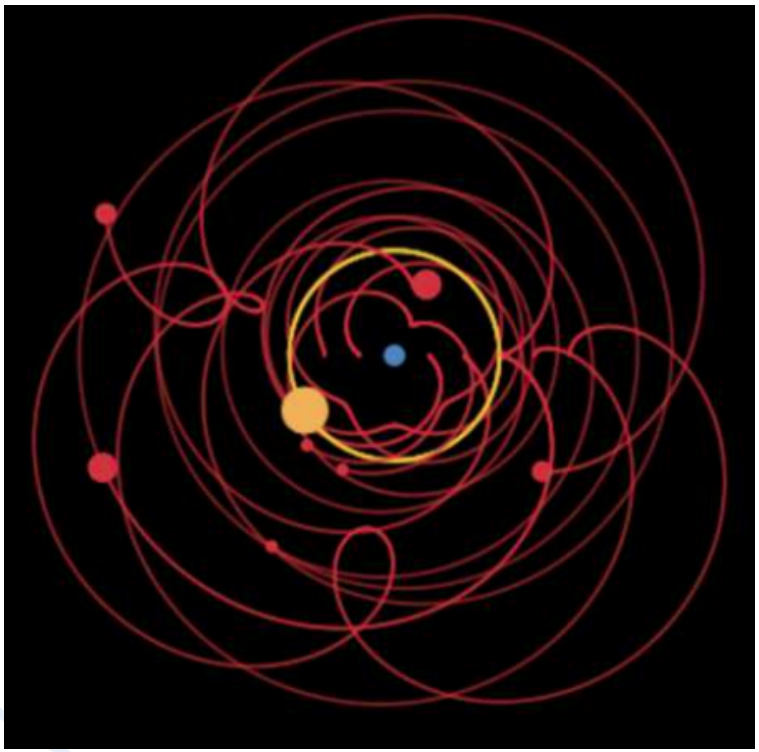
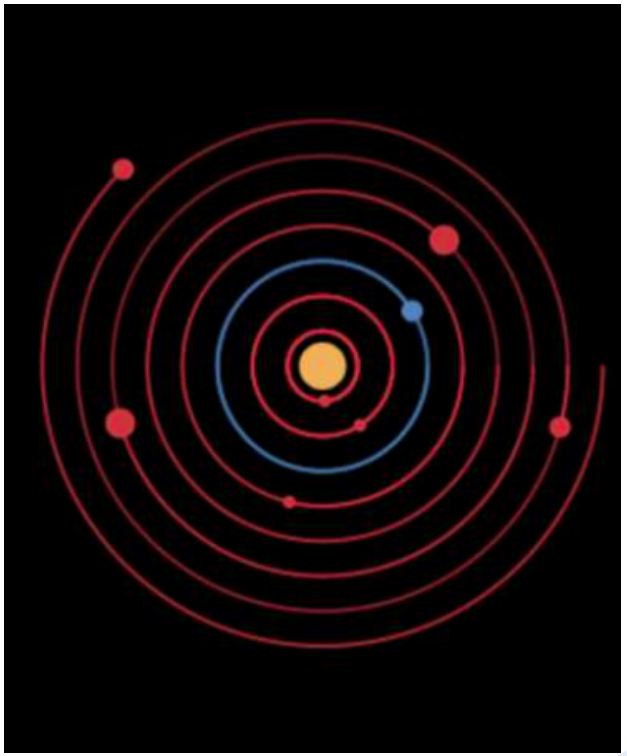
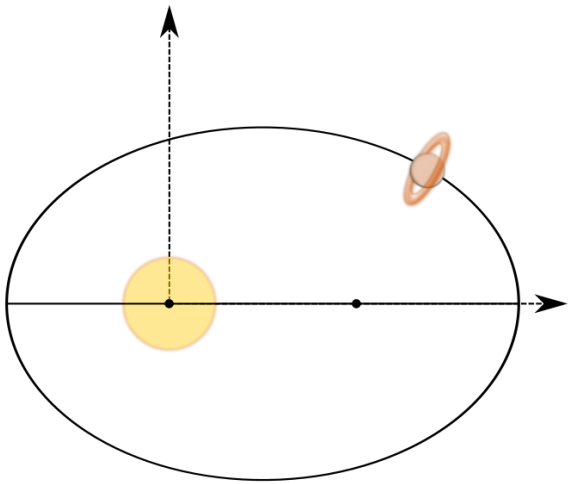


Heliocentric



Historical introduction : Kepler

1. Elliptical orbits



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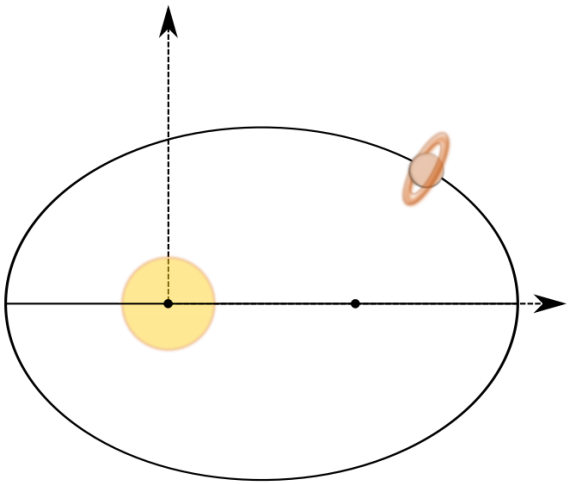
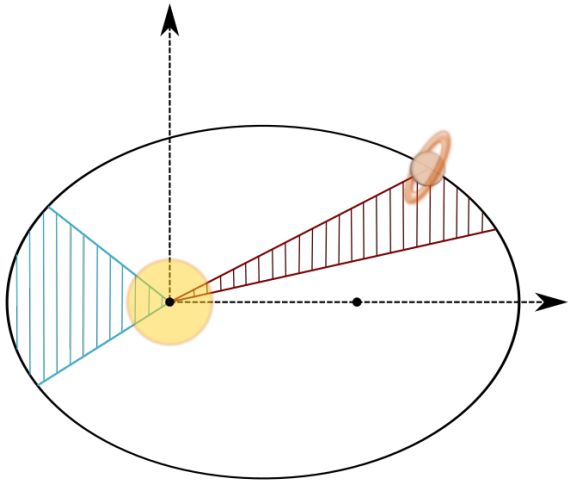


2. Angular velocity



Equal areas in equal times

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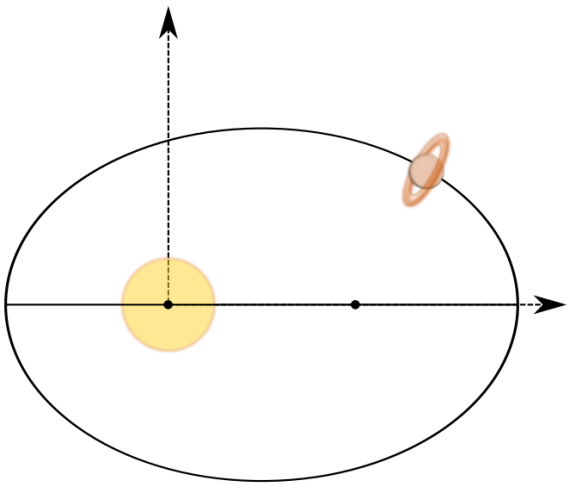

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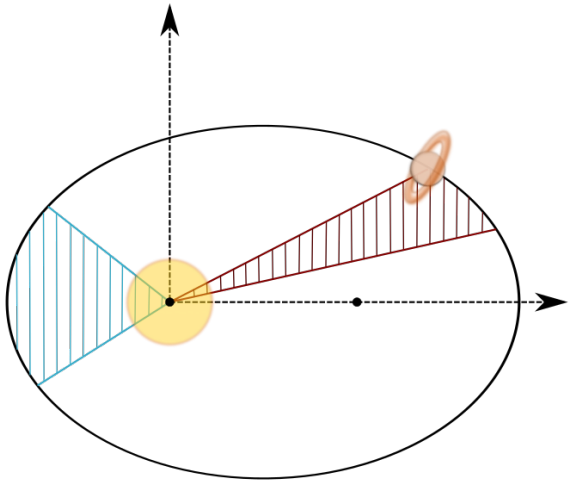
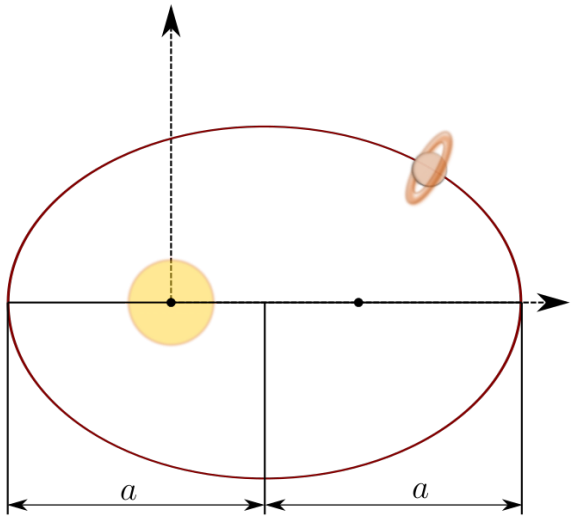


Equal areas in equal times

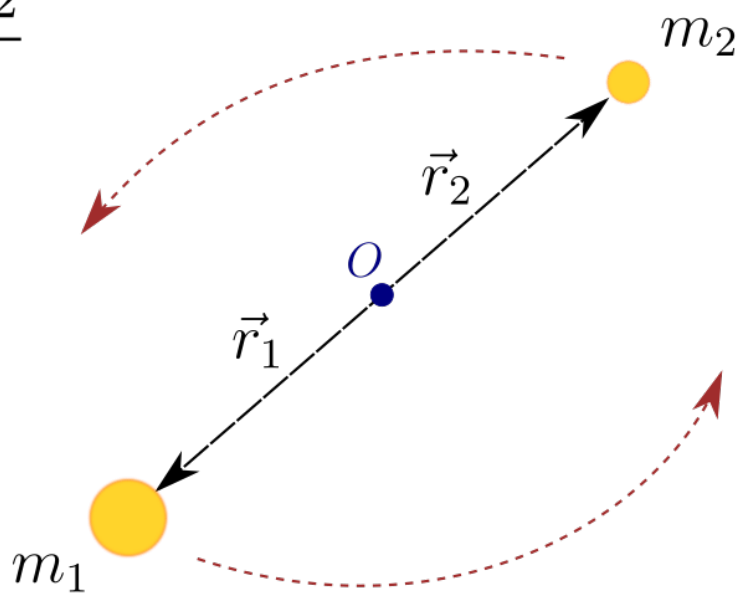
3. Period of the orbit

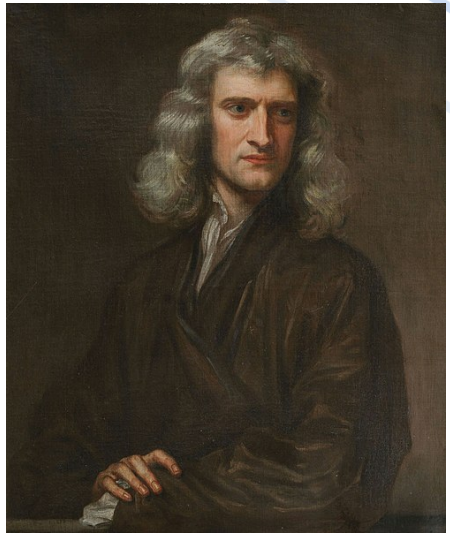


$T^2 \propto a^3$

Historical introduction : Newton

$$\vec{F} = G\frac{m_1 m_2}{\vec{r}_{12}^{\,2}}$$





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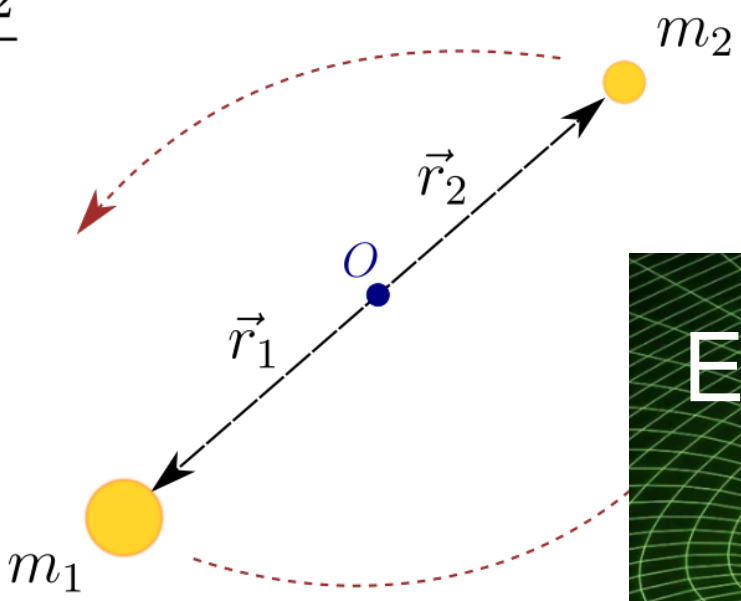

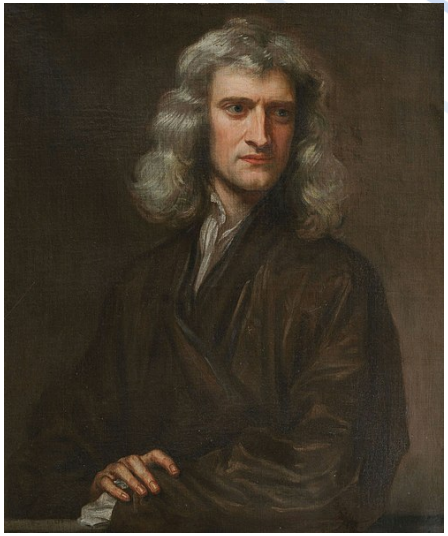

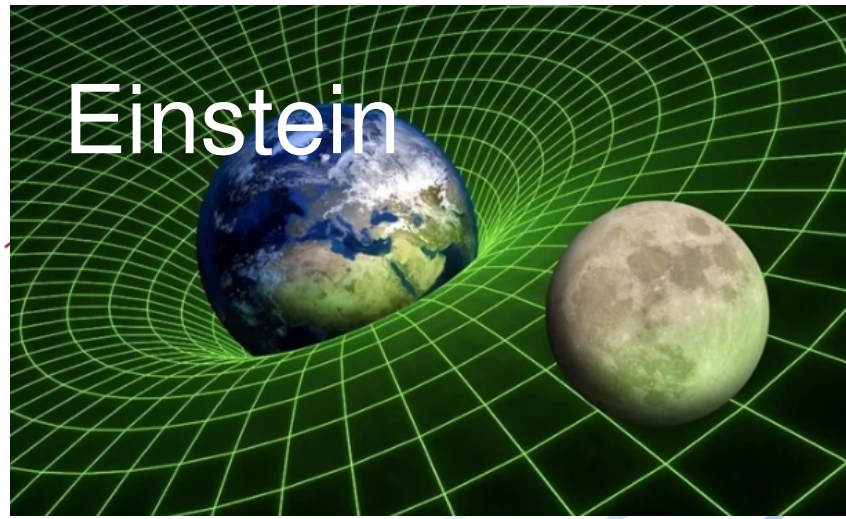

Image Credit : canbedone/Shutterstock.com

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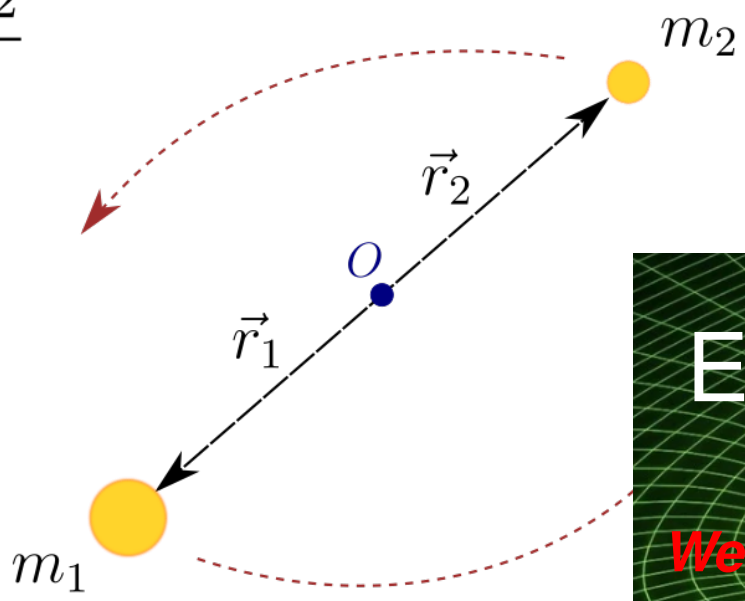
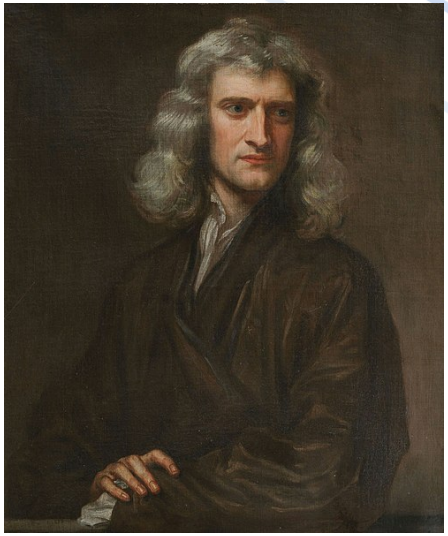




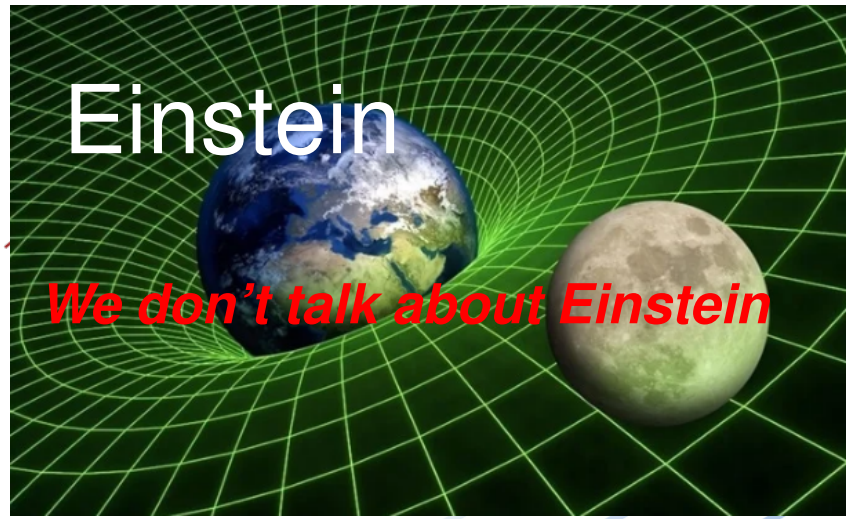


Image Credit : canbedone/Shutterstock.com

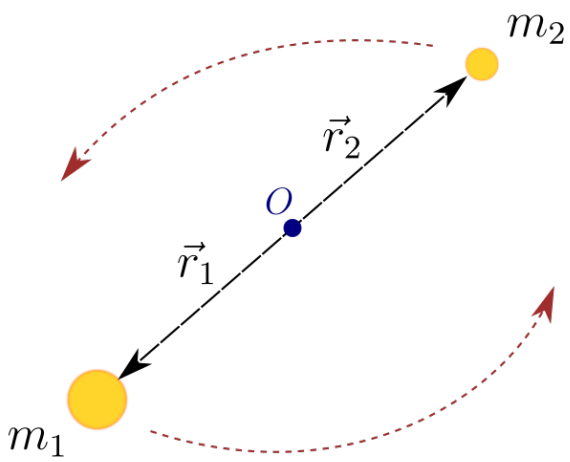
What is the 3-body problem ?

(dramatic music plays in the background)

Quick summary

2-body problem

1. We can solve it analytically
2. We can predict it



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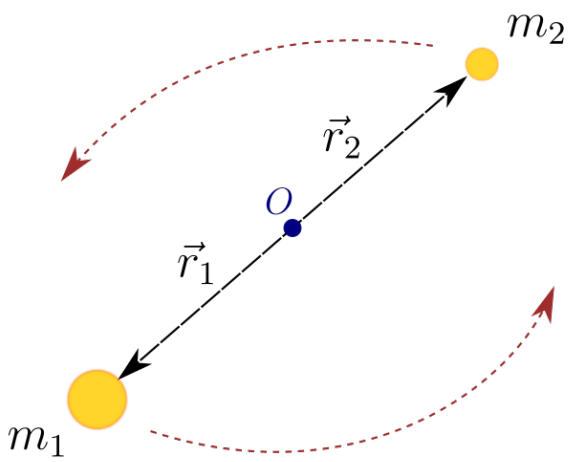
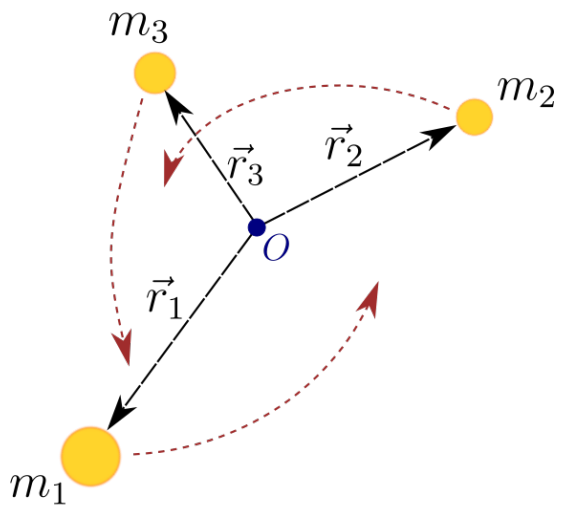


3-body problem

1. We have to solve it numerically
2. It is chaotic



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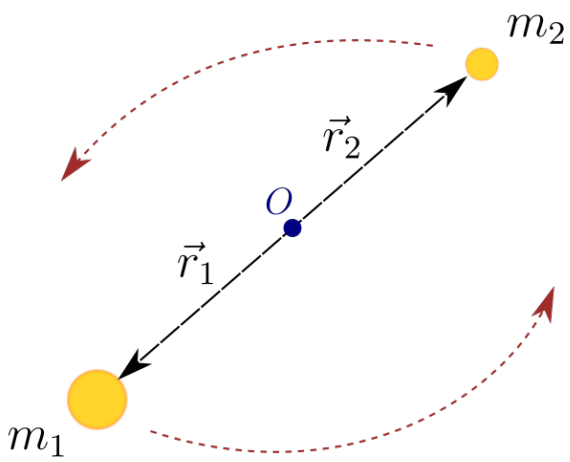

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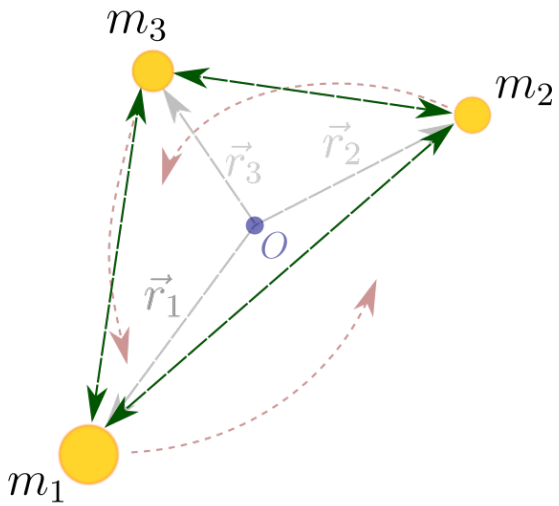

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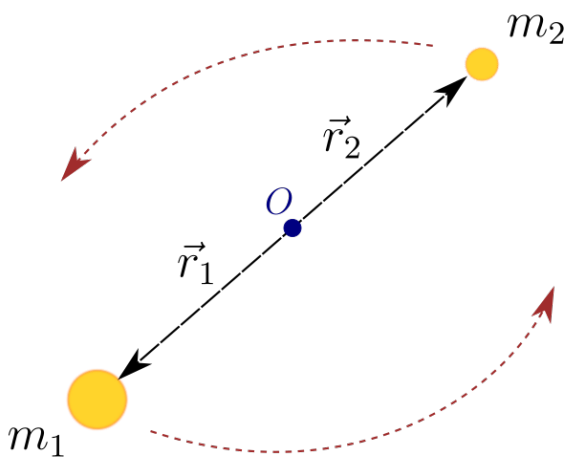

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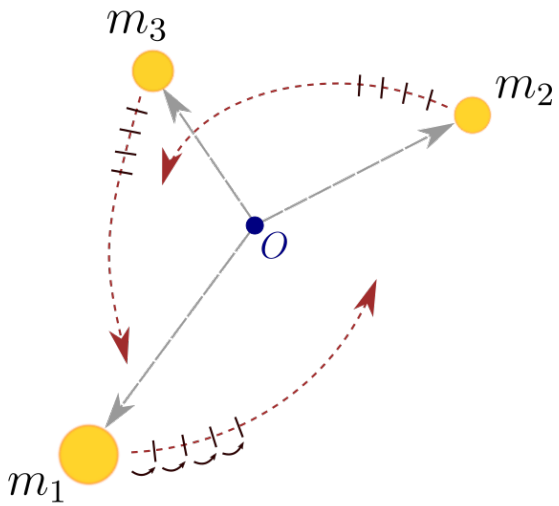

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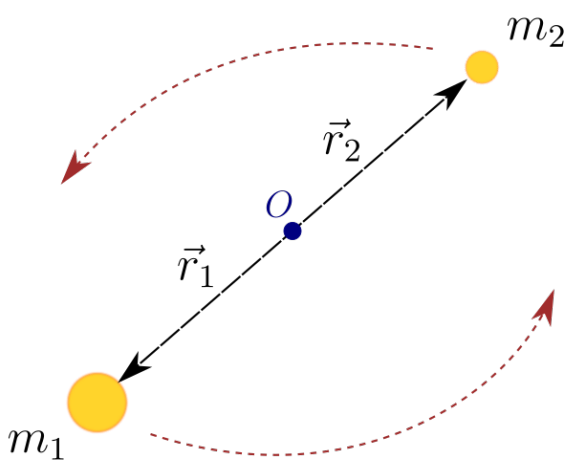

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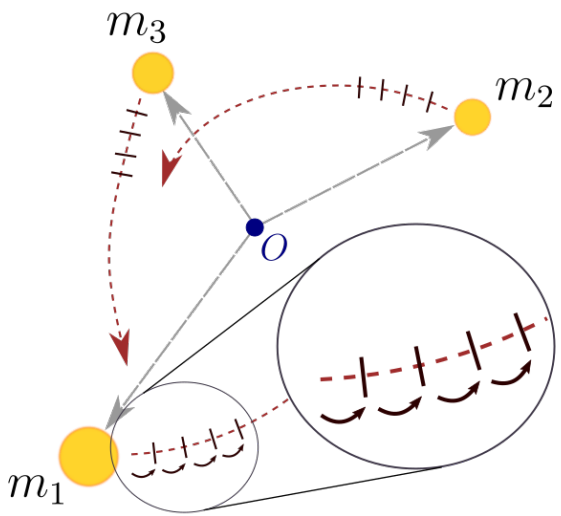

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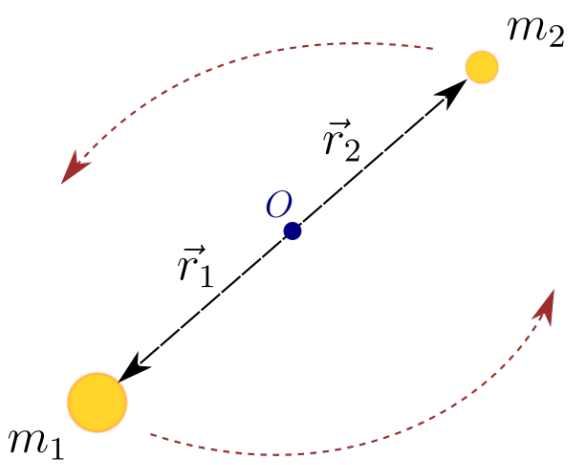

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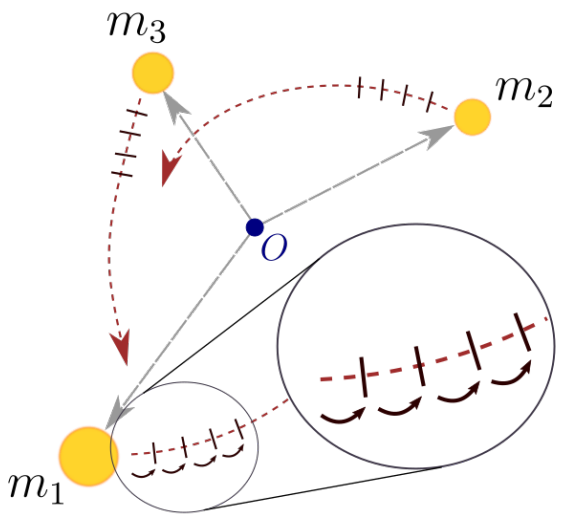
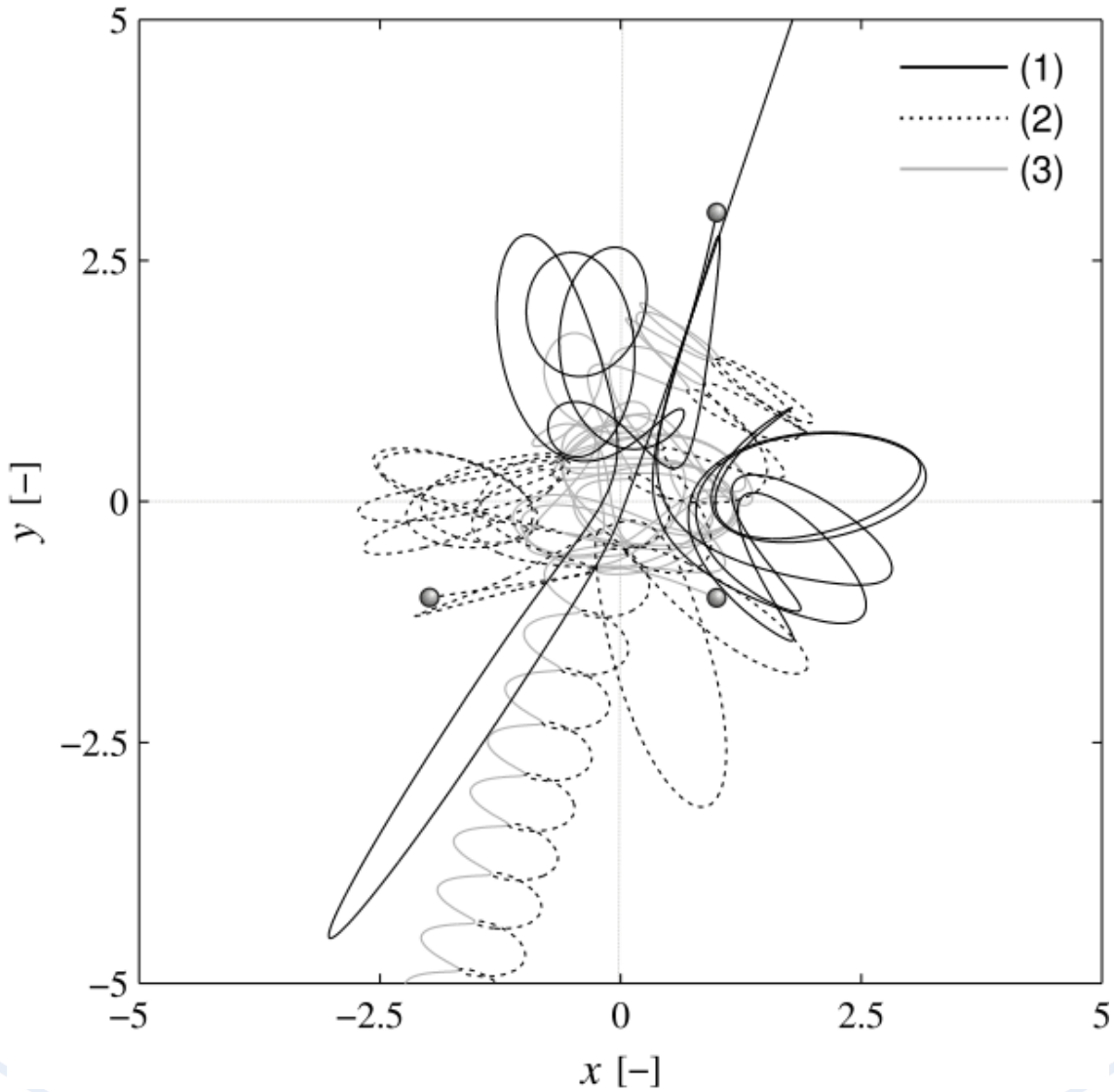
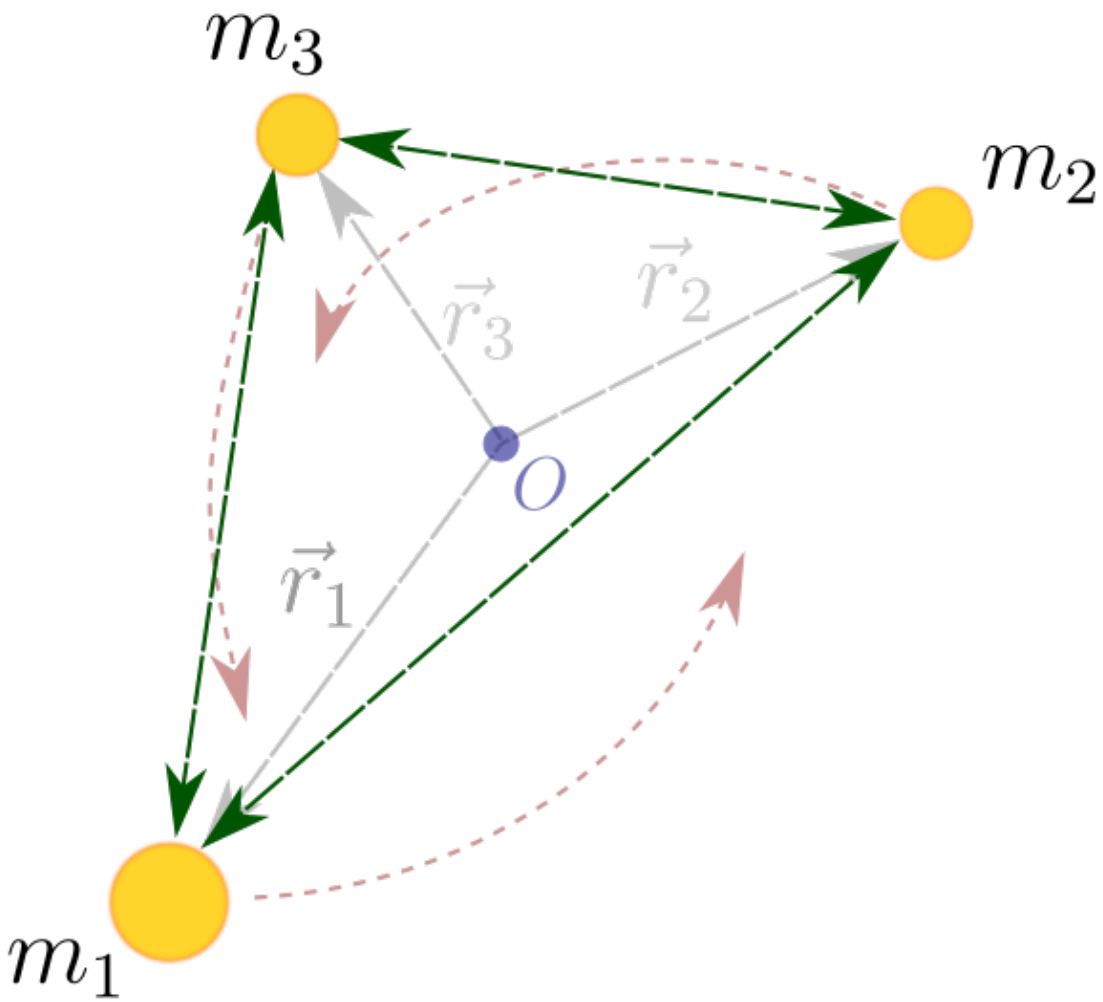


Sun-Earth-Moon

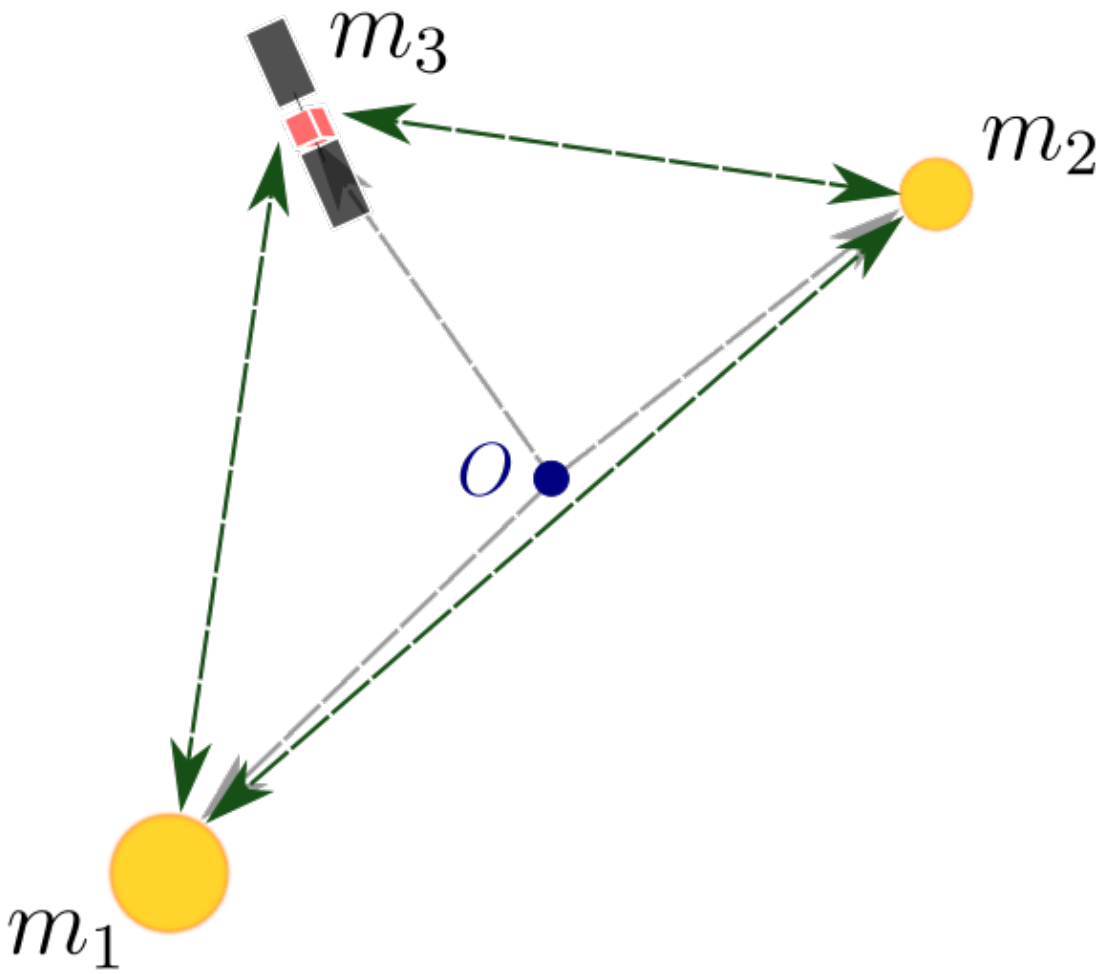
Chaos



A special case...



A special case...



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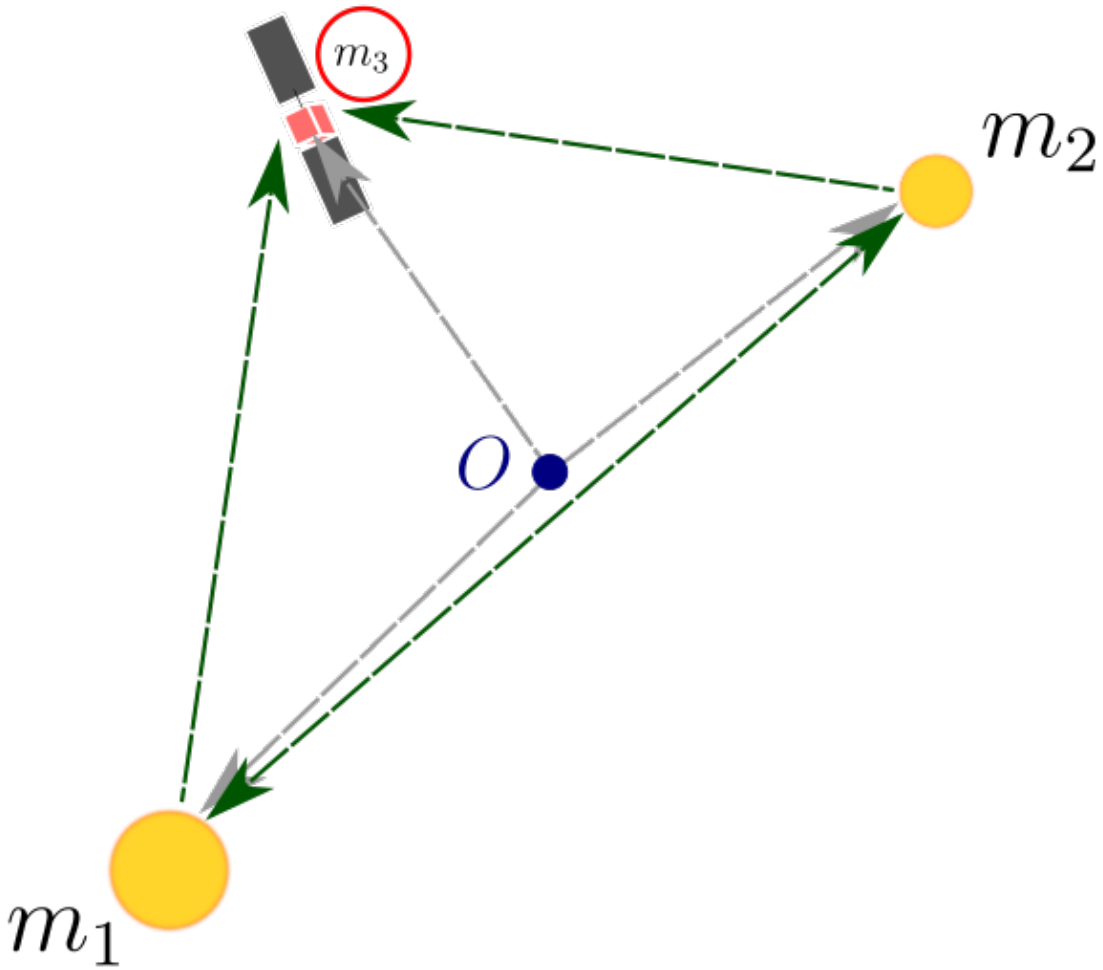

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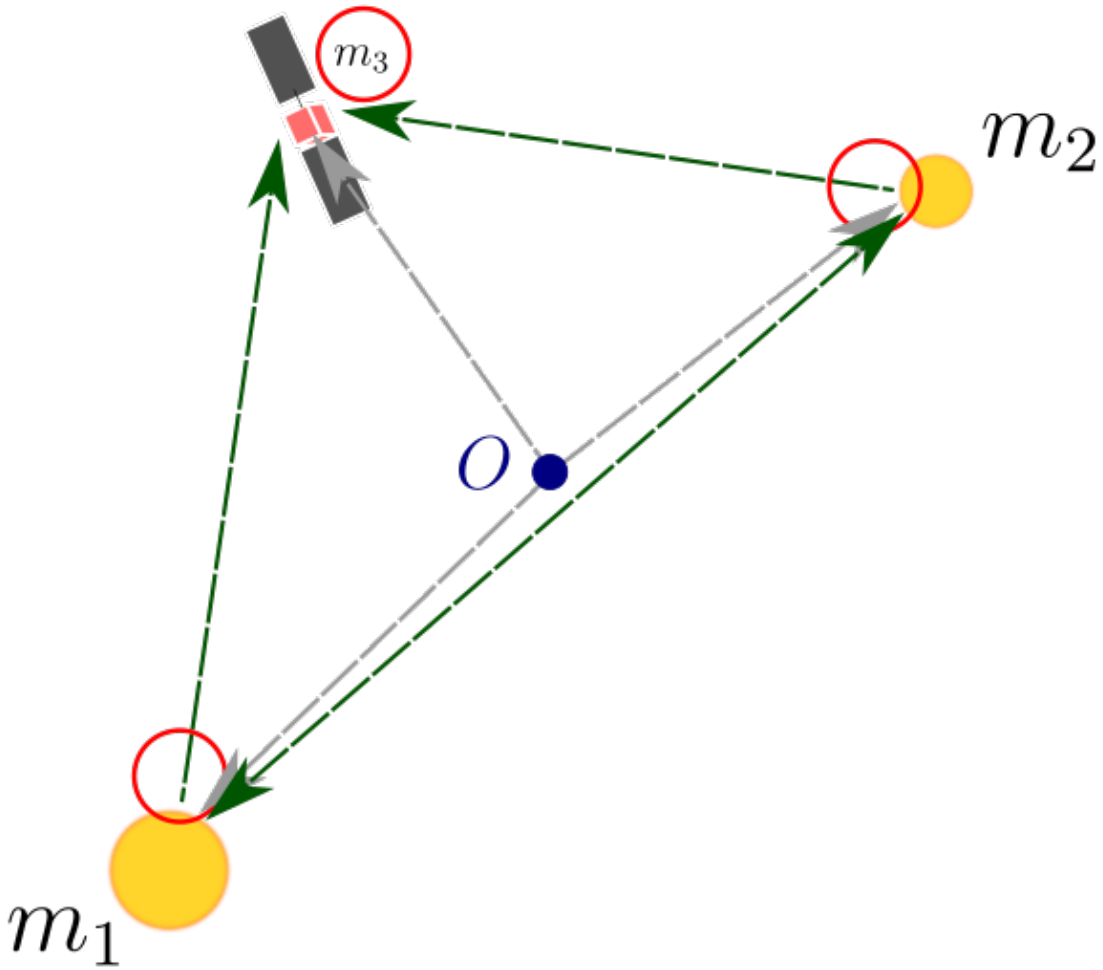
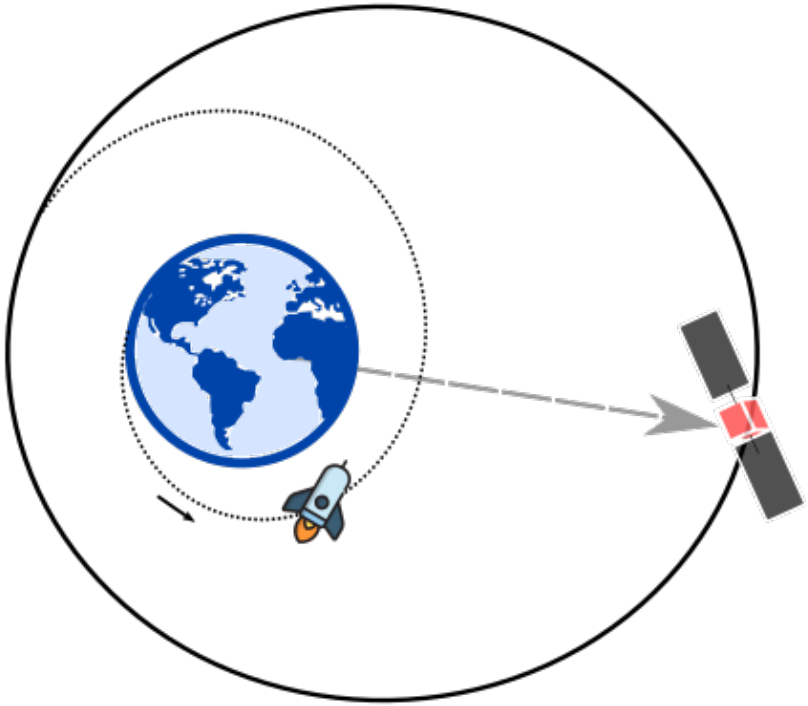


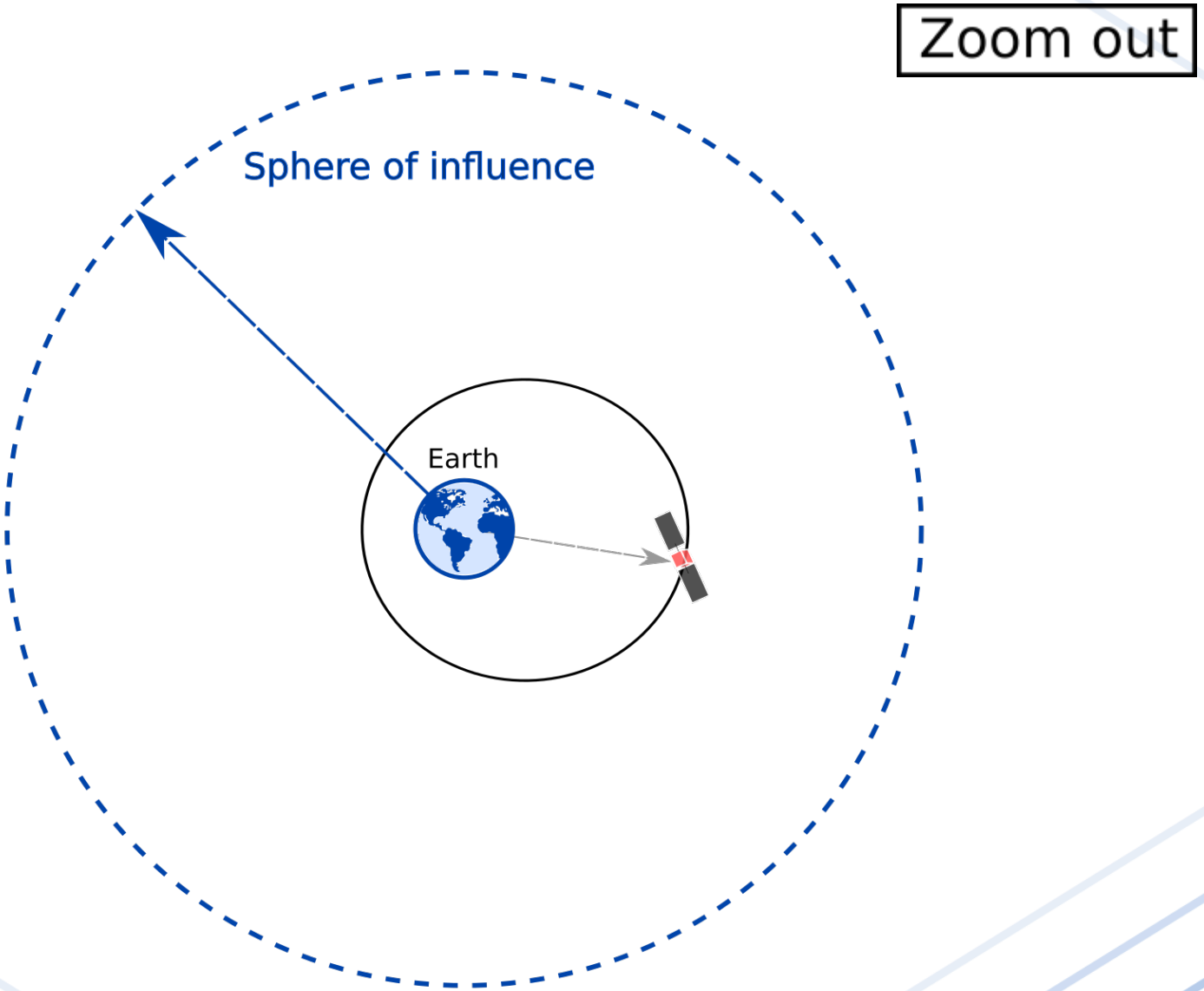
How do we get to other planets ?

Starting point : The Earth

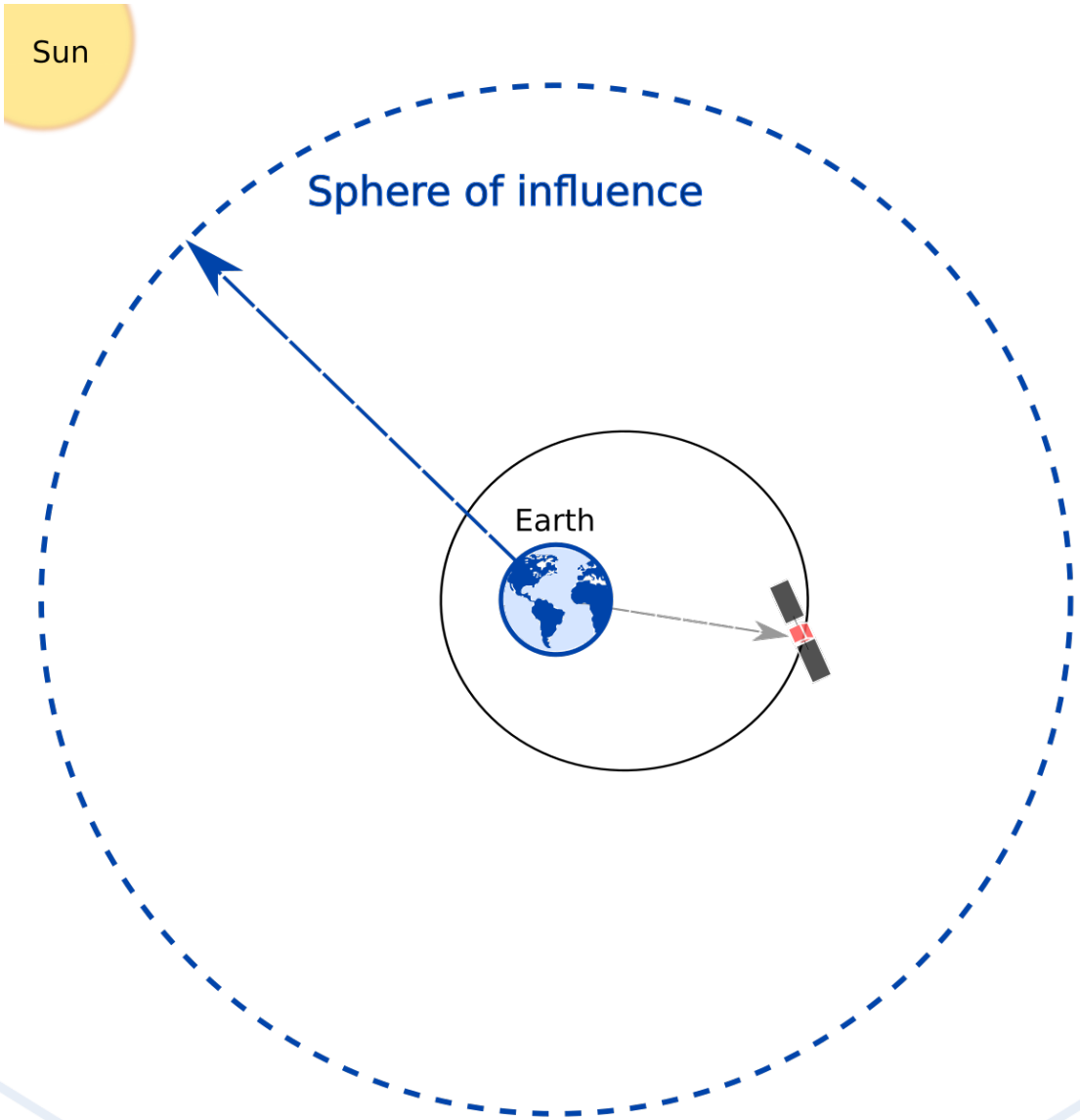
Zoom in



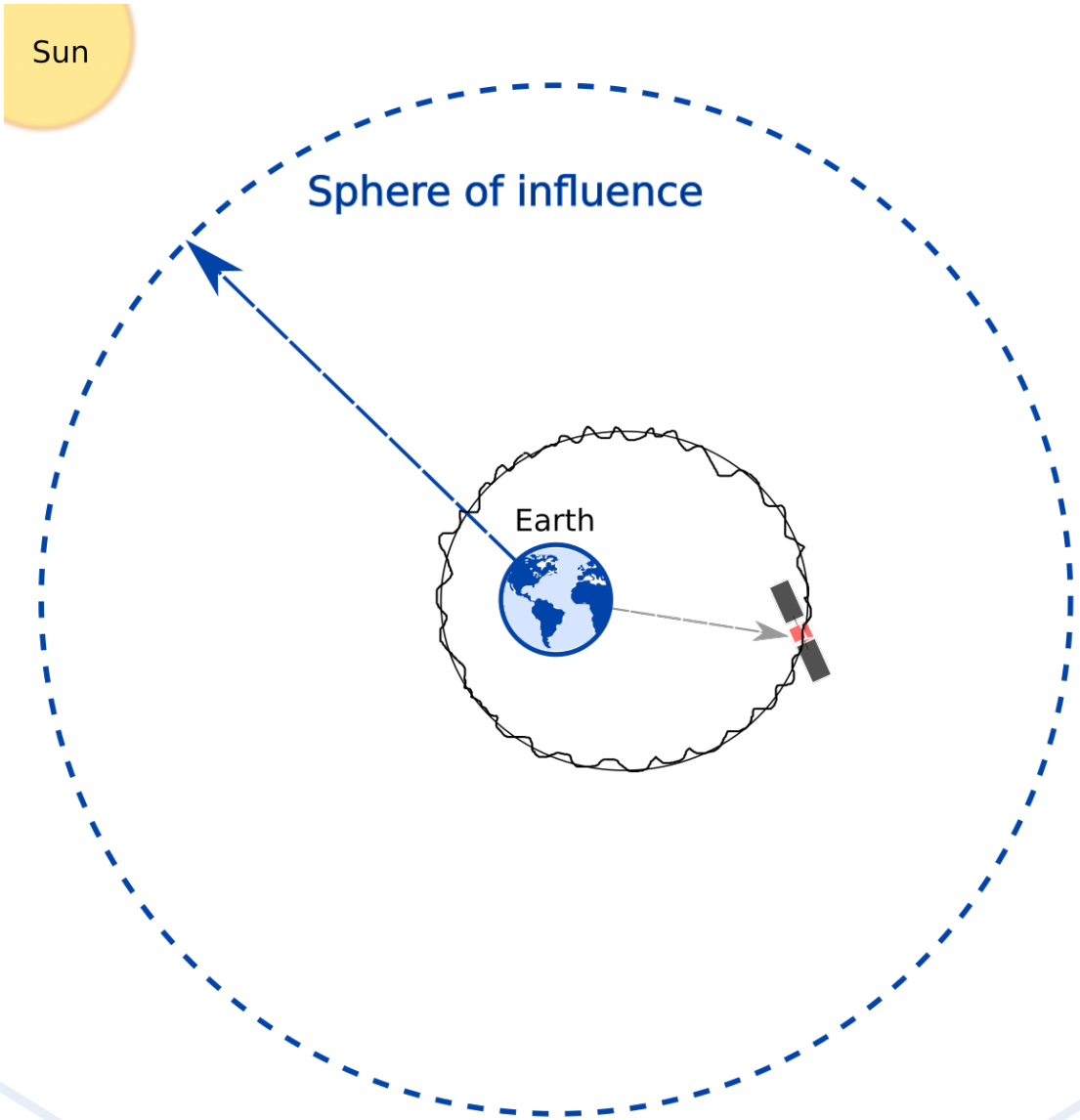
Starting point : The Earth



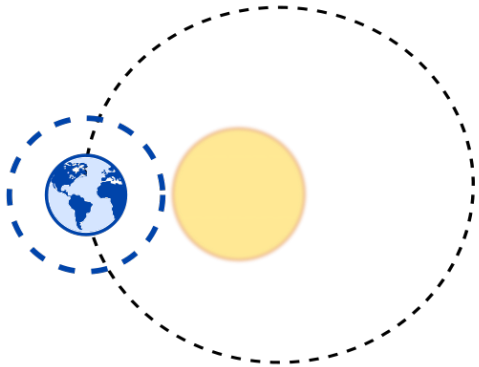
Starting point : The Earth



Starting point : The Earth



The interplanetary trip



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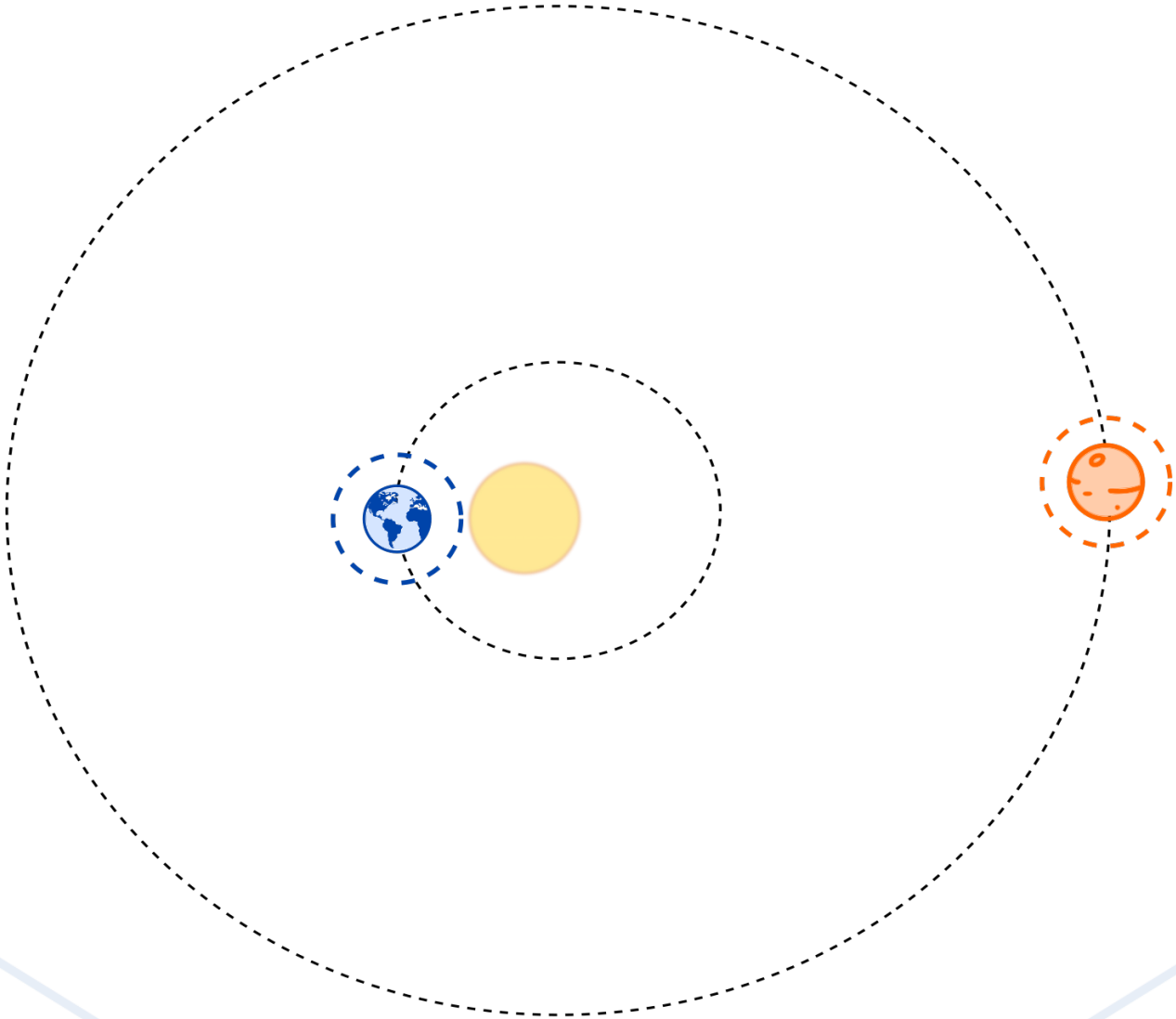

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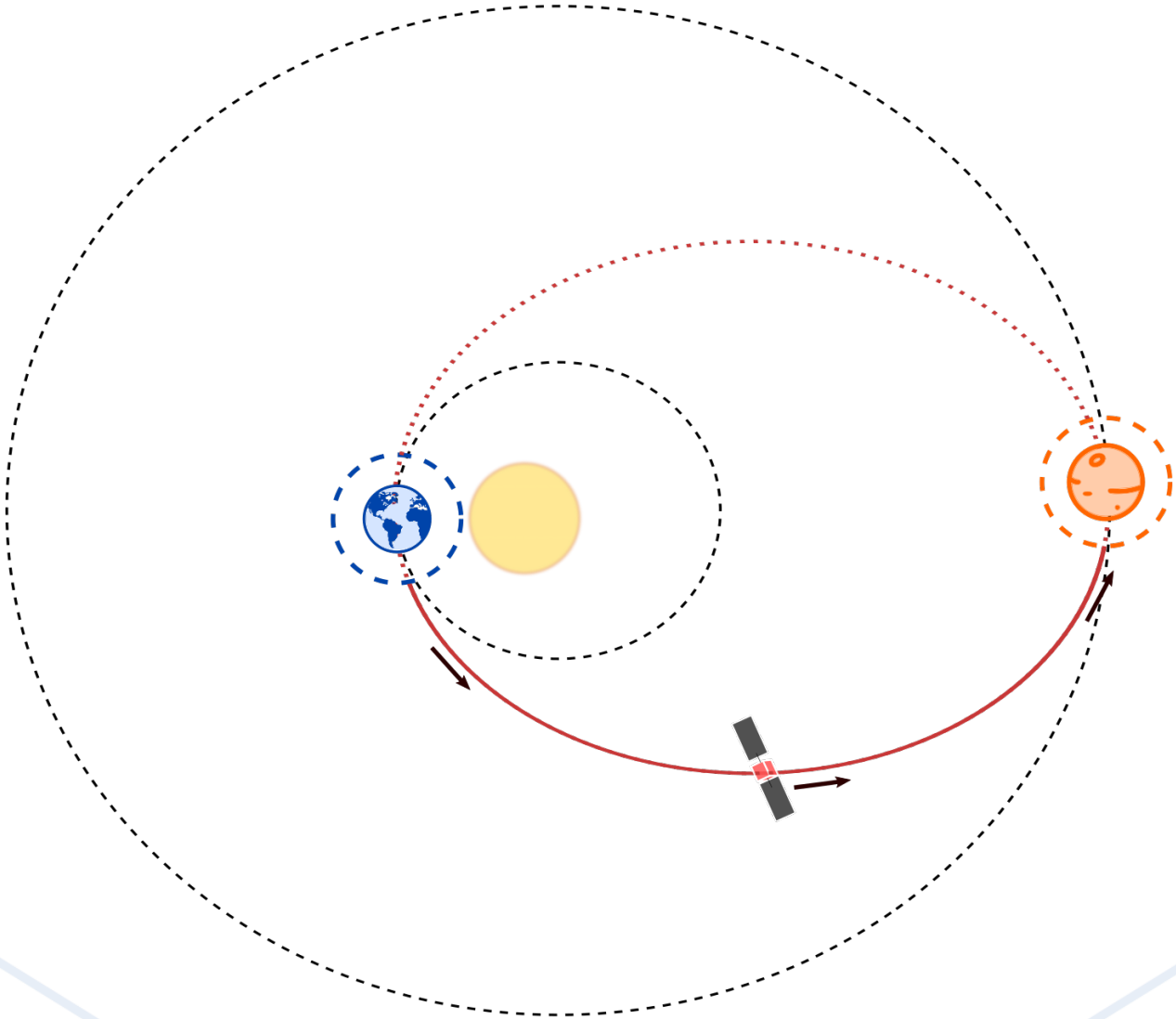
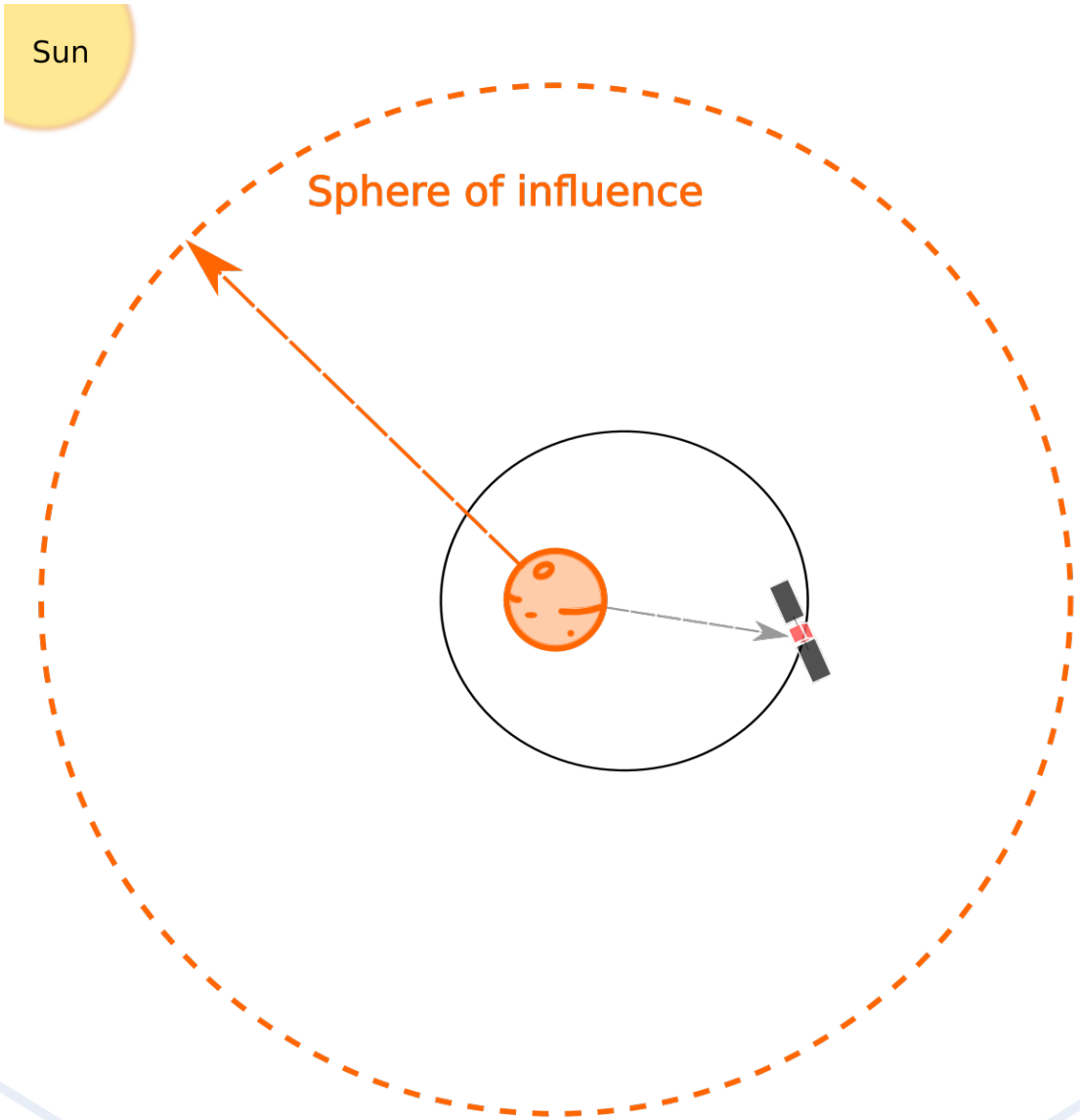
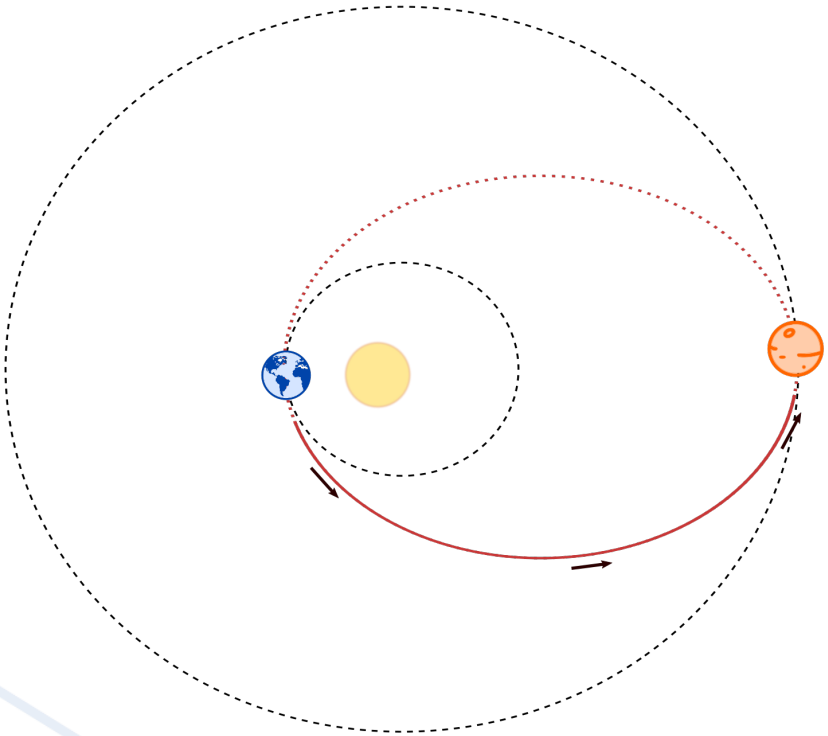


Arrival



Going to the outer Solar System



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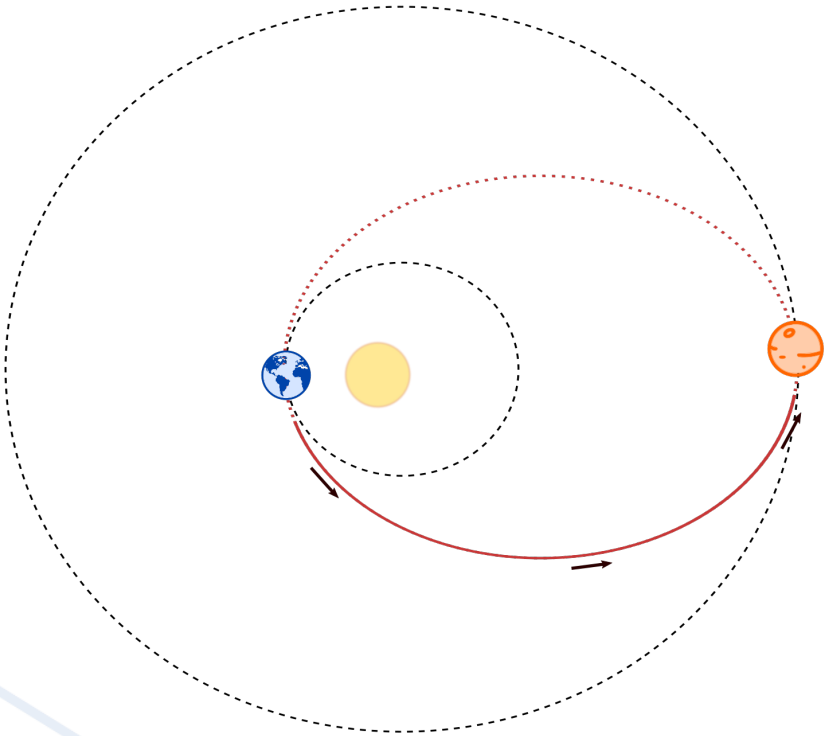

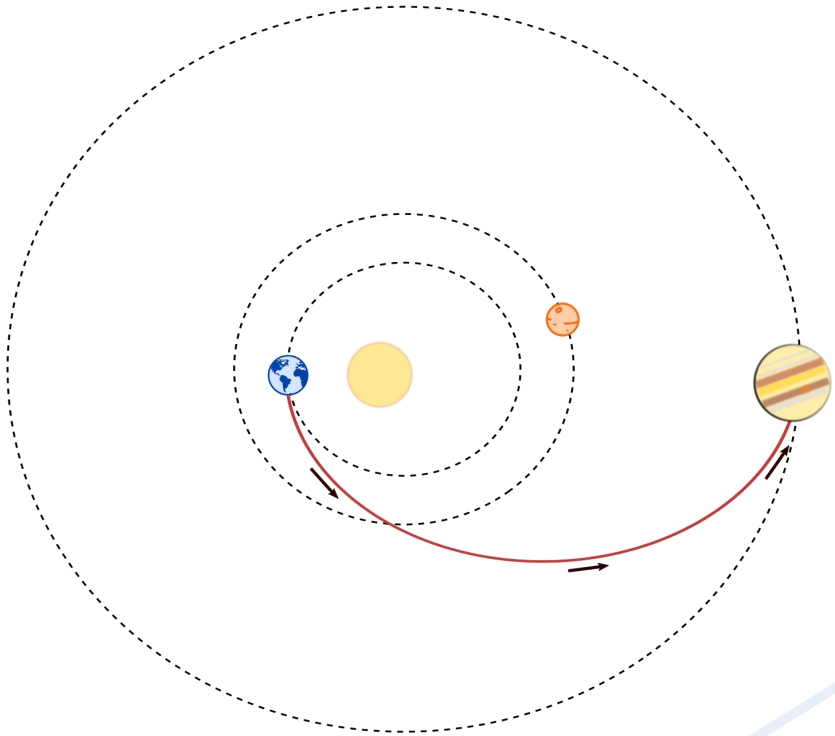

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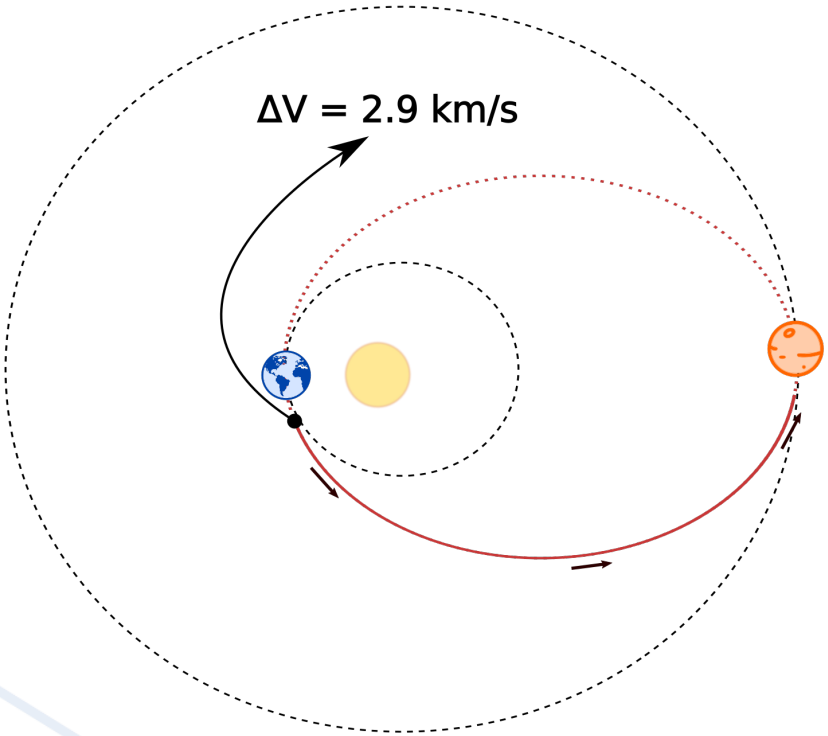

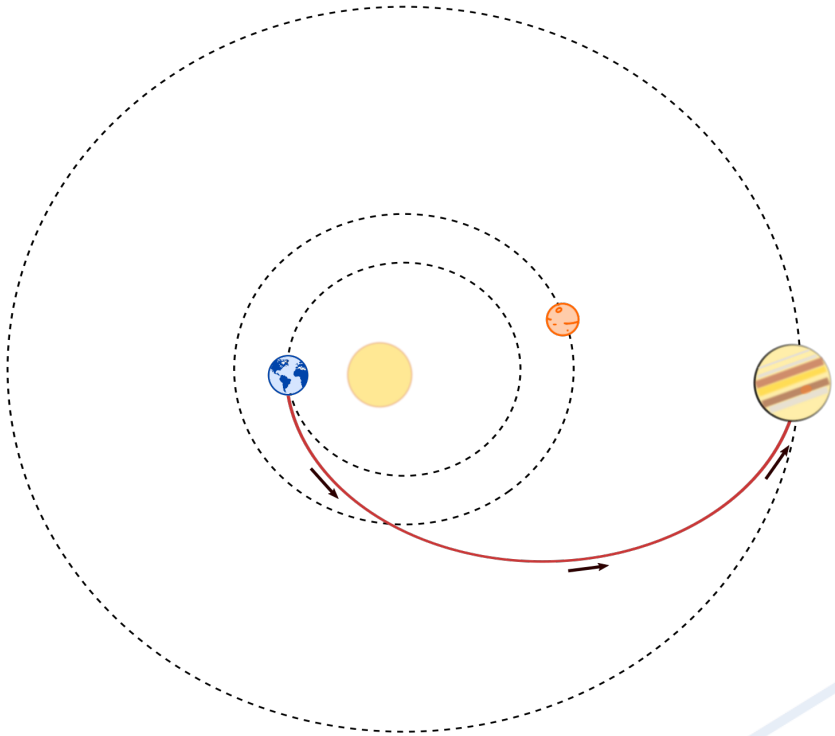

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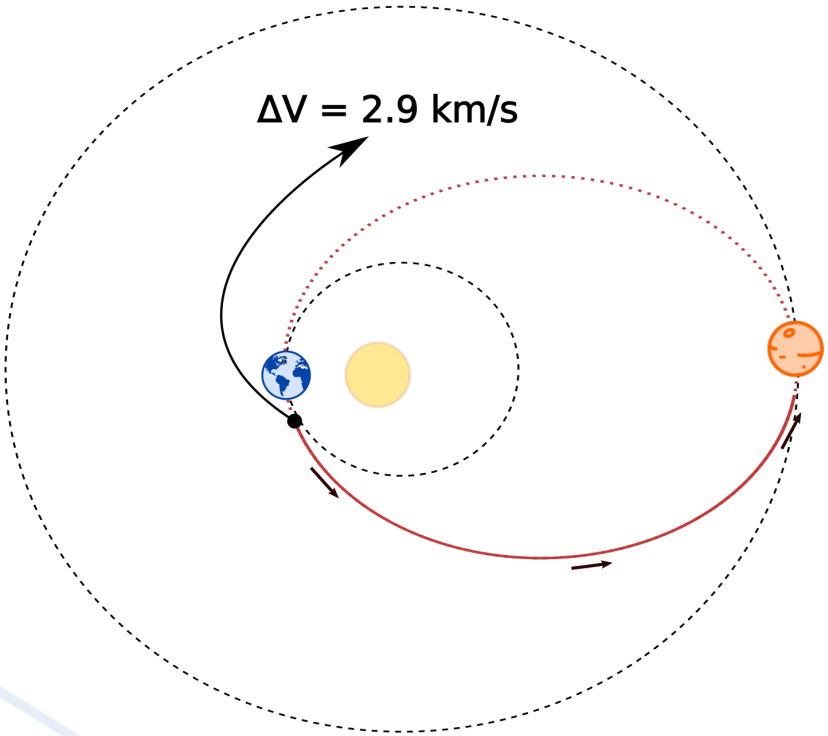
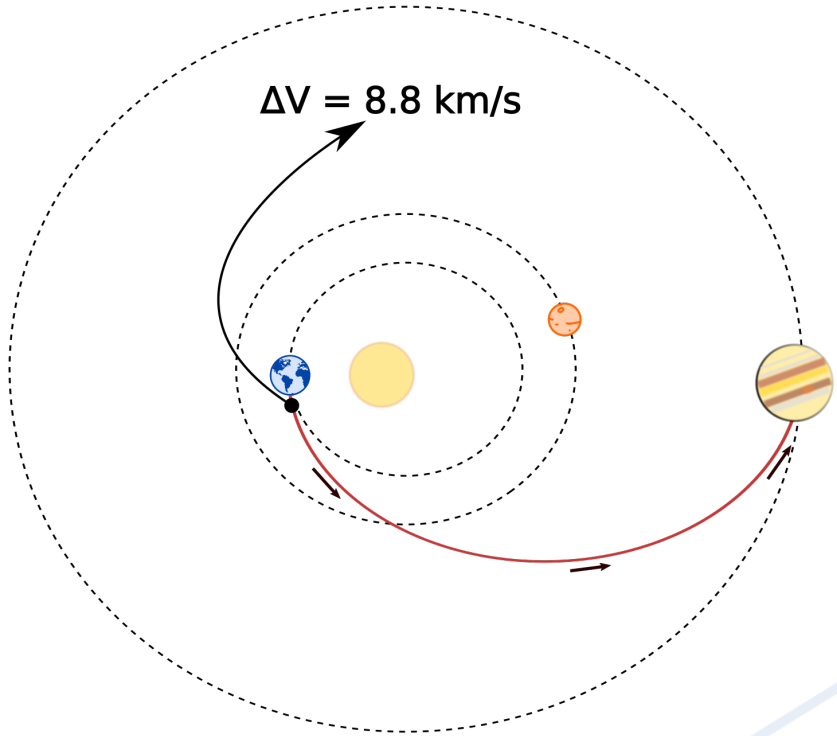
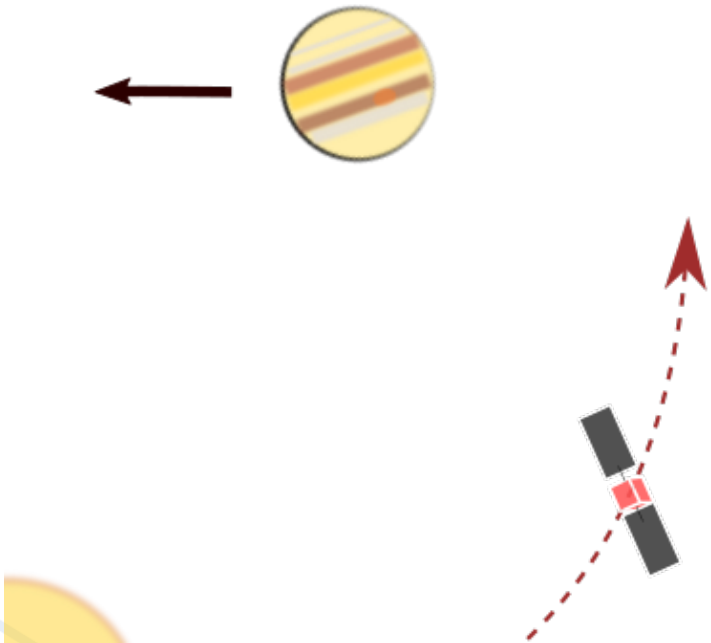




Gravitational Assists (slingshot)

Speed-up



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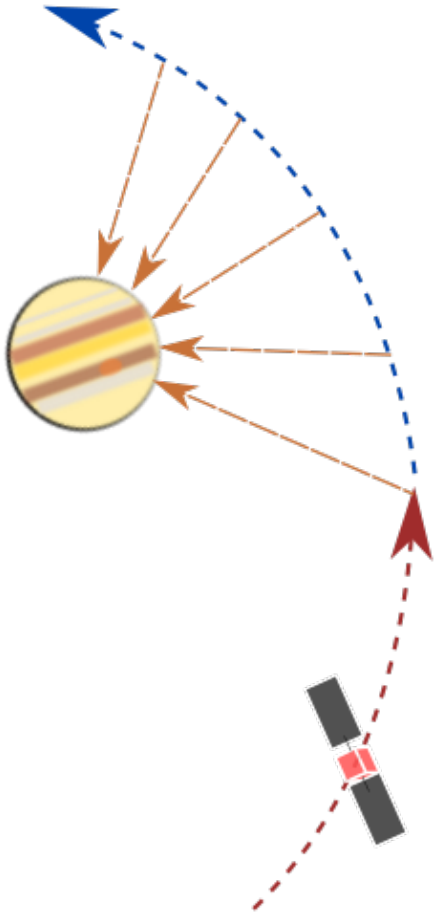

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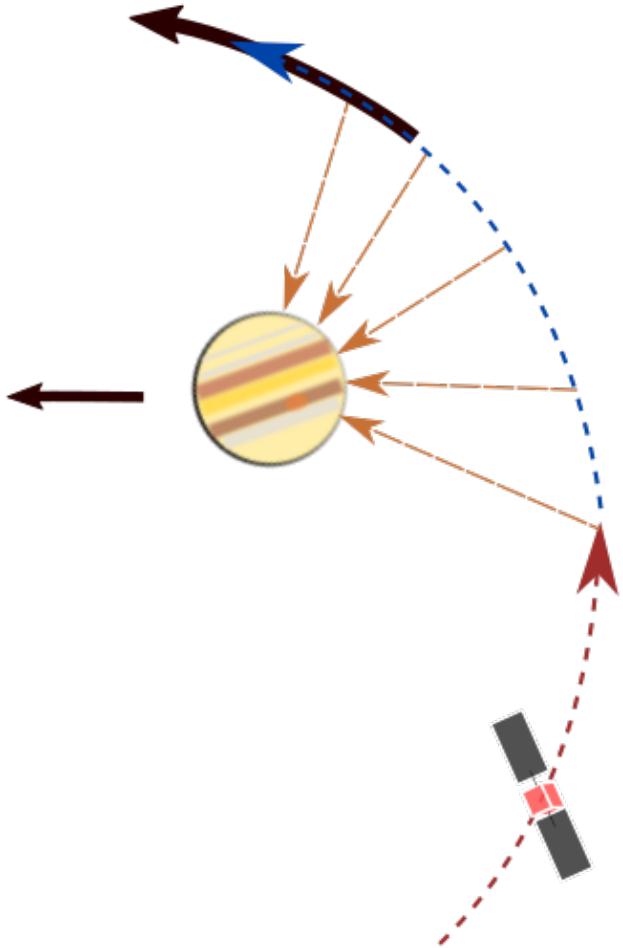
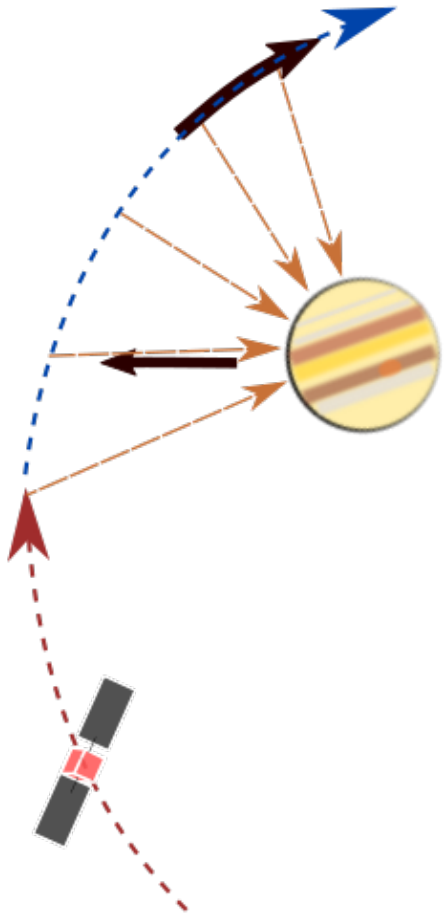
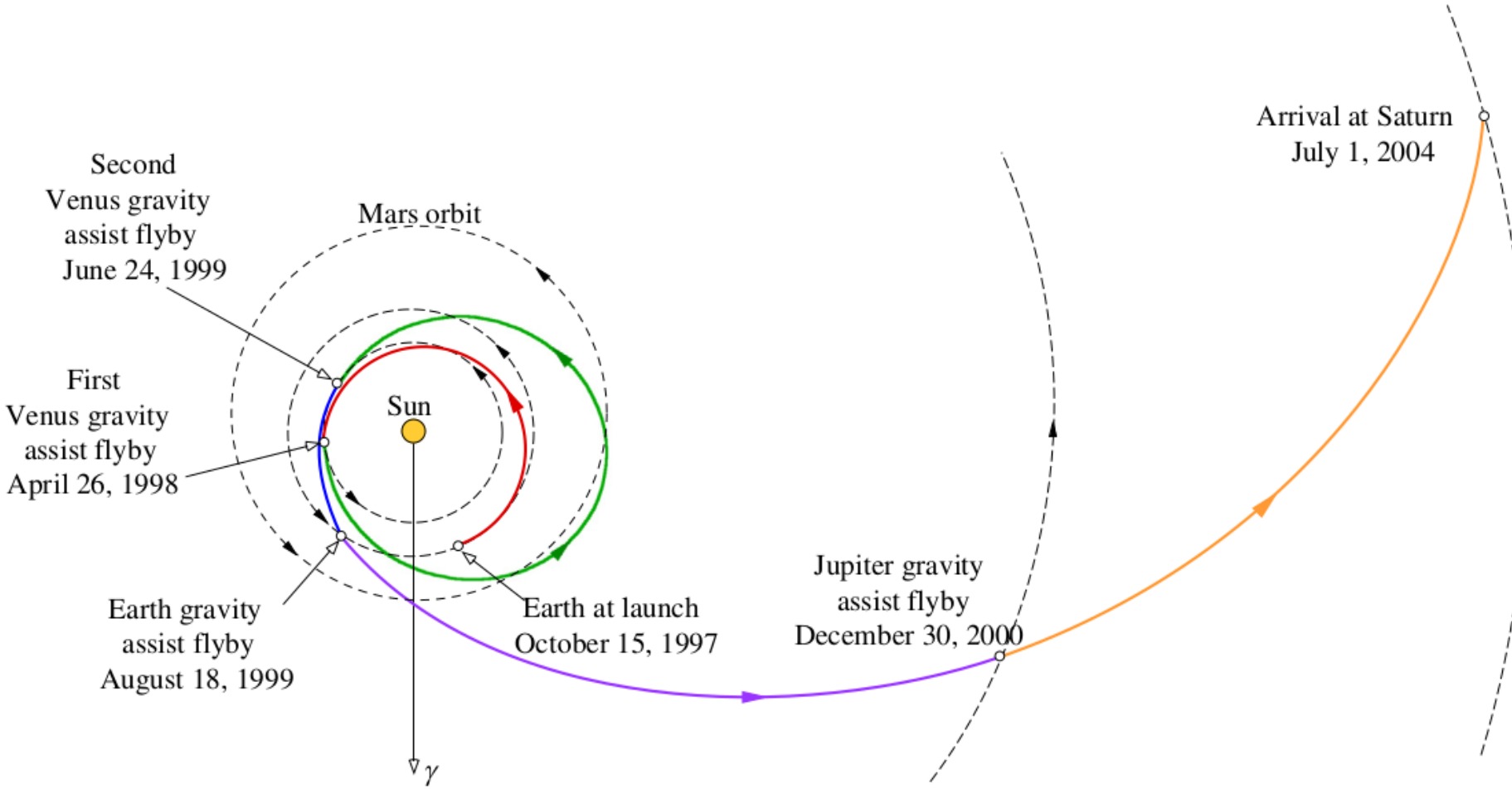


Gravitational Assists (slingshot)

Decelerate

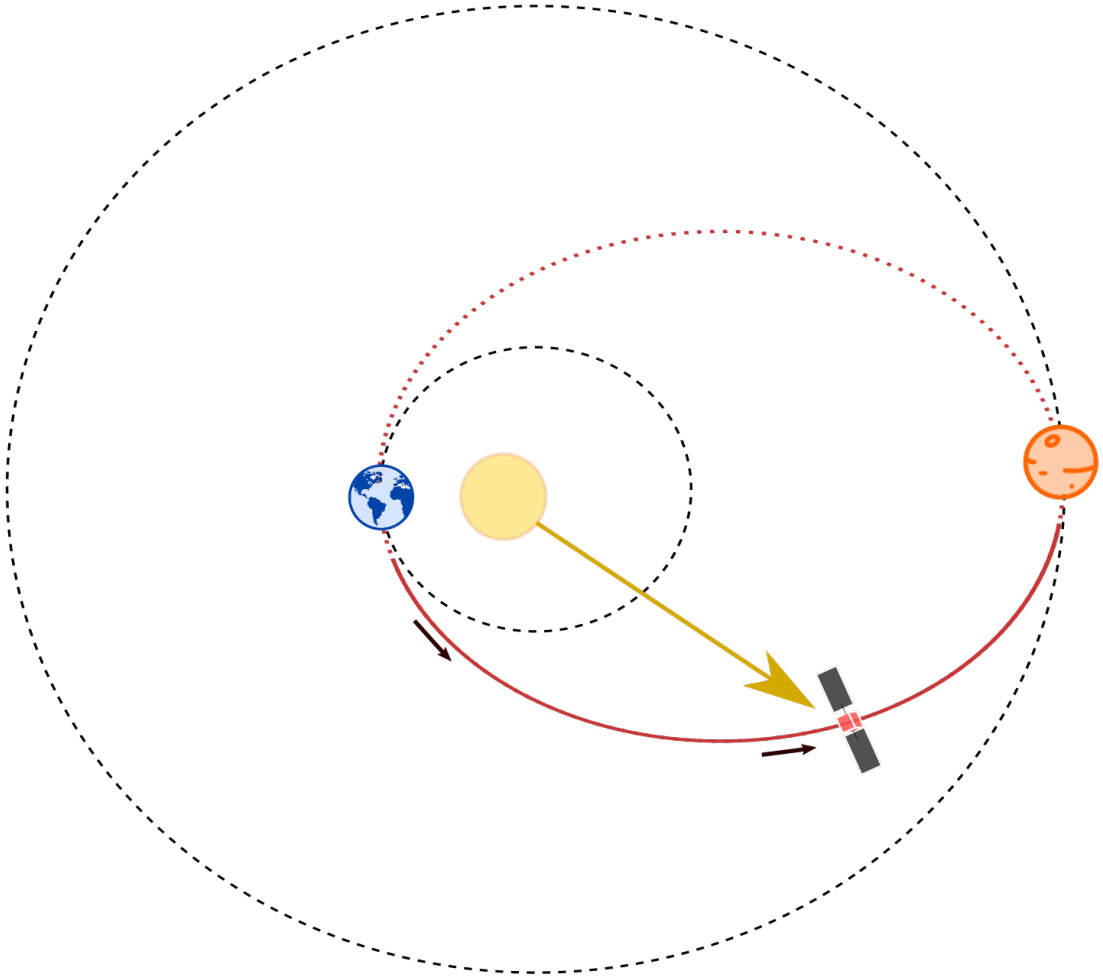


A real transfer : the orbit of Cassini



Source : Howard Curtis, Orbital Mechanics for Engineering Students

The reality of interplanetary transfers



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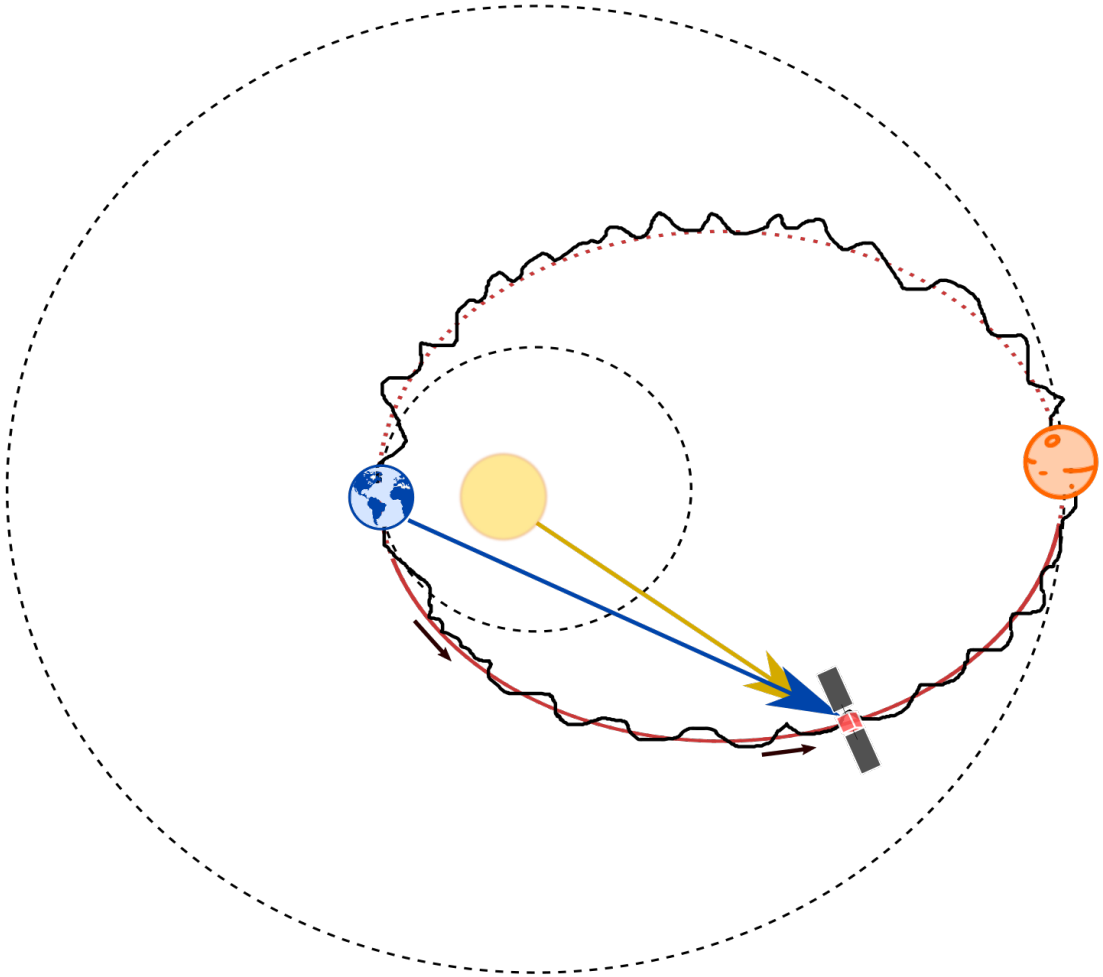

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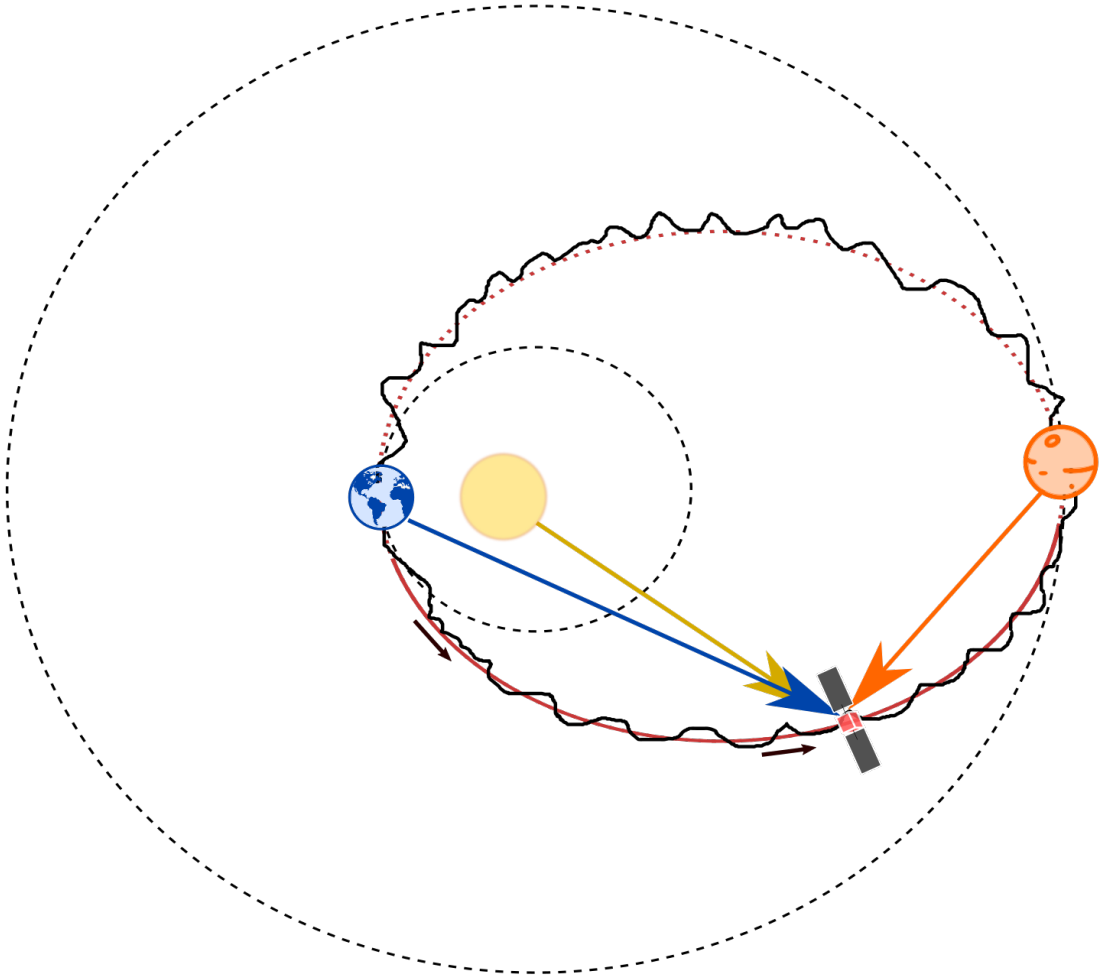
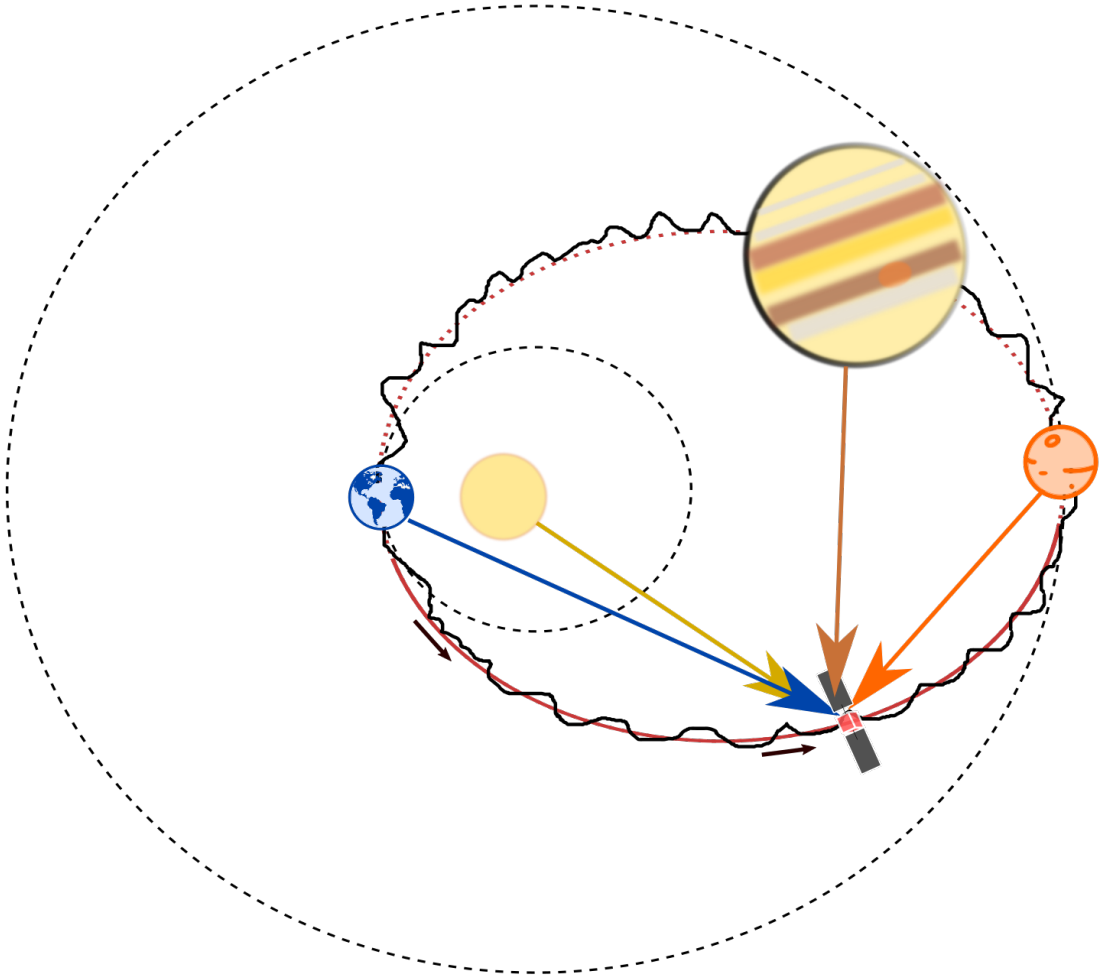


The reality of interplanetary transfers



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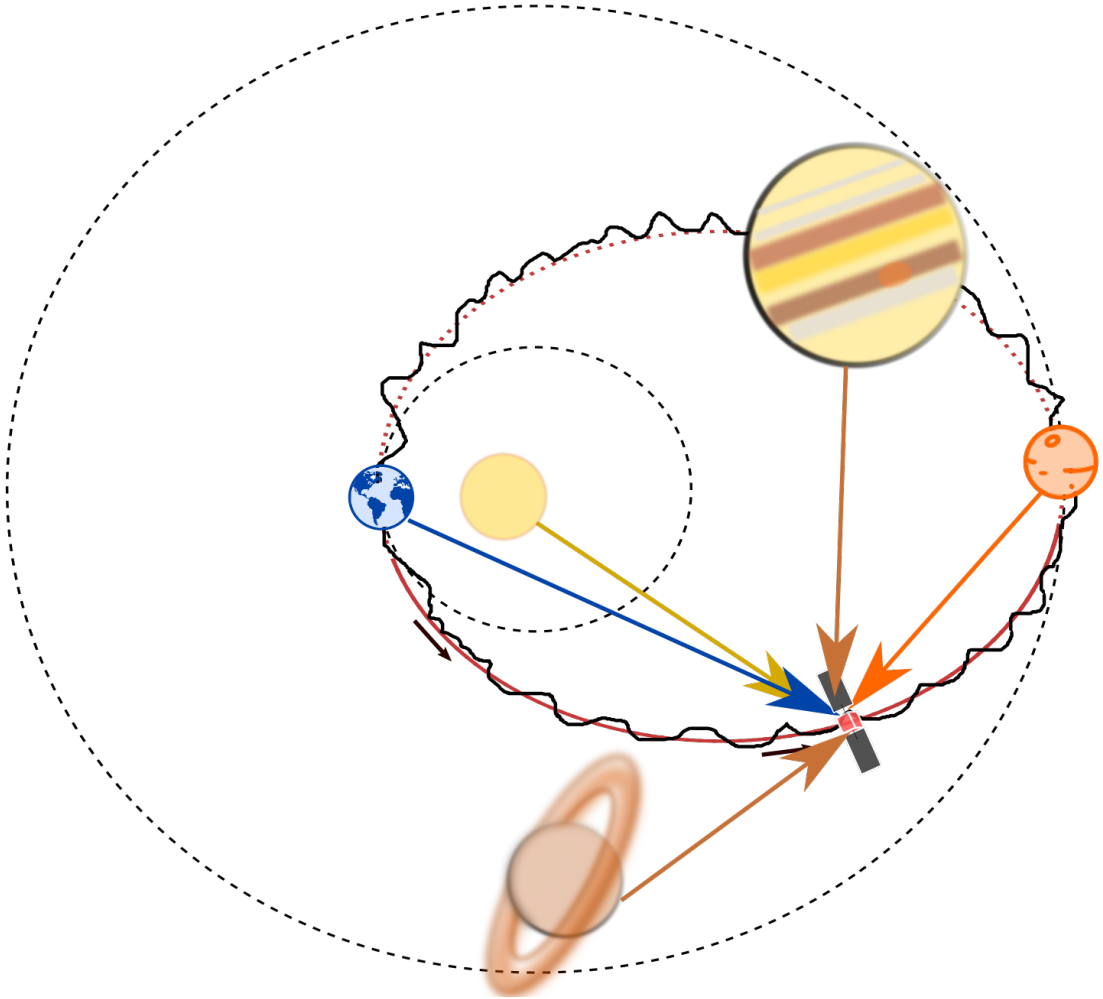

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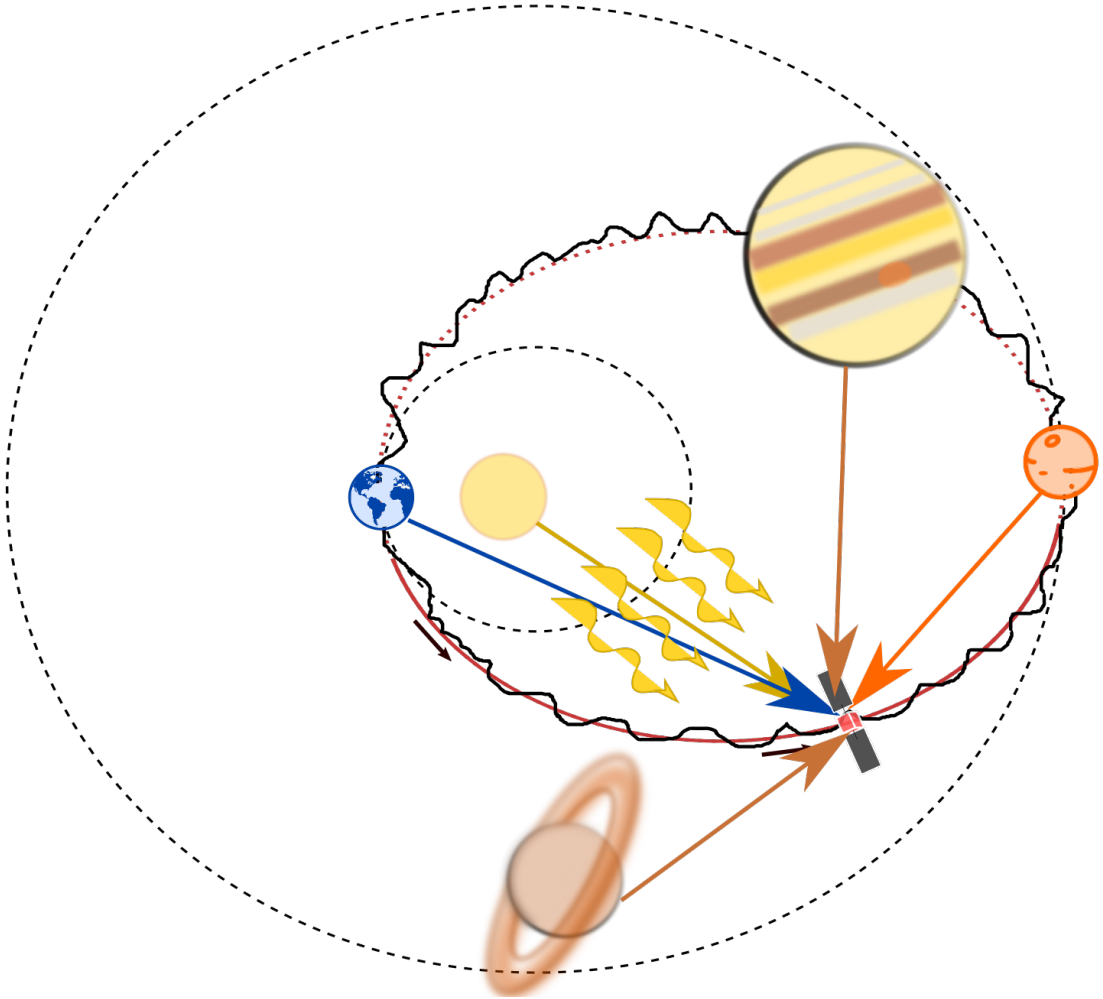

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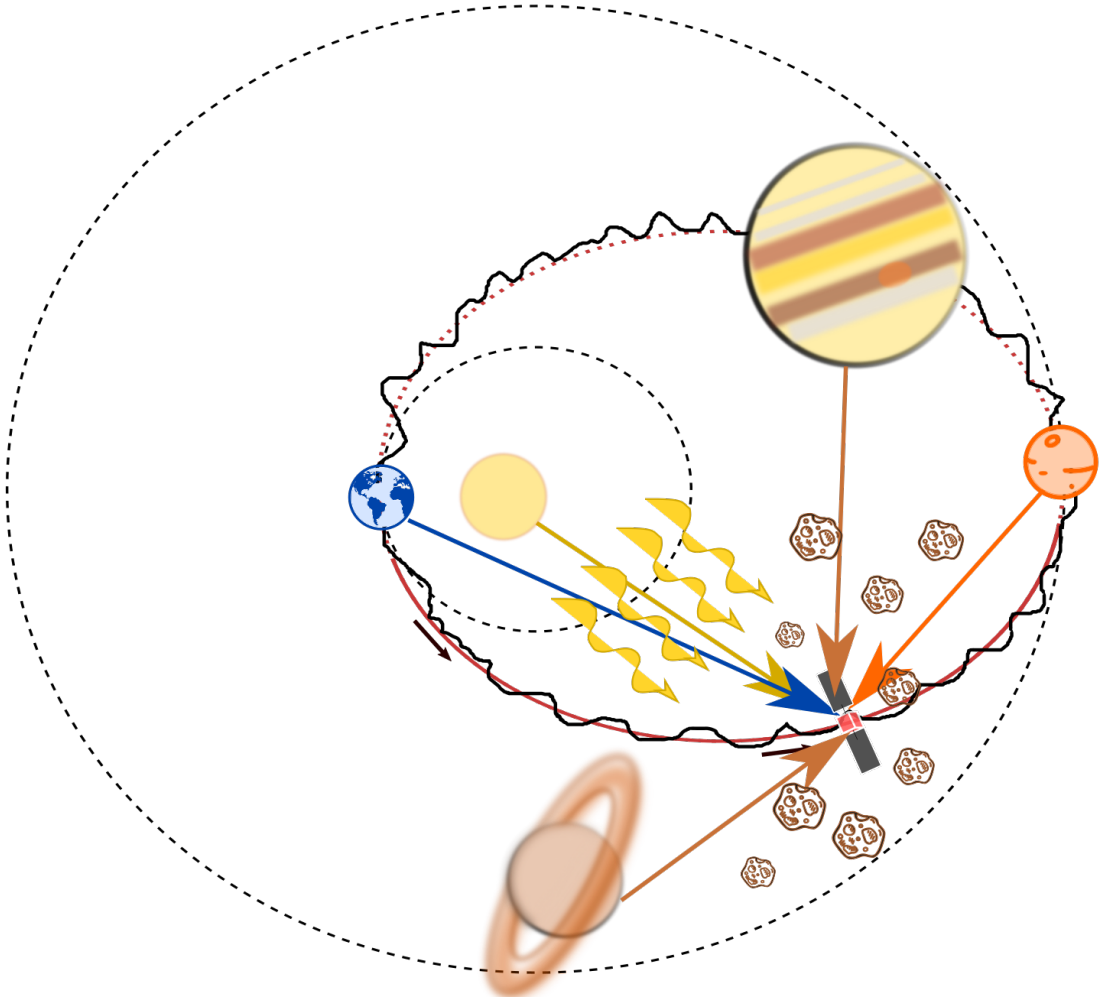

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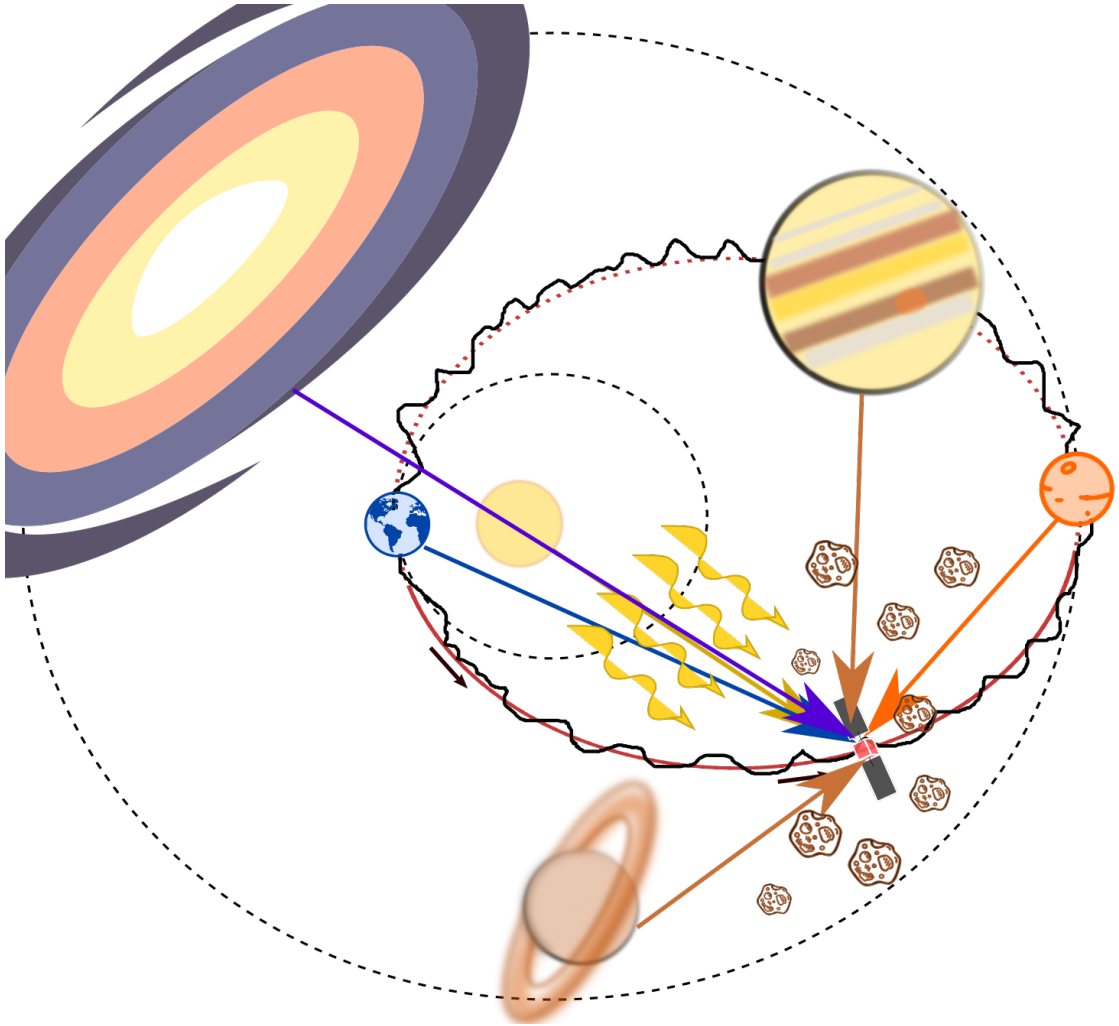

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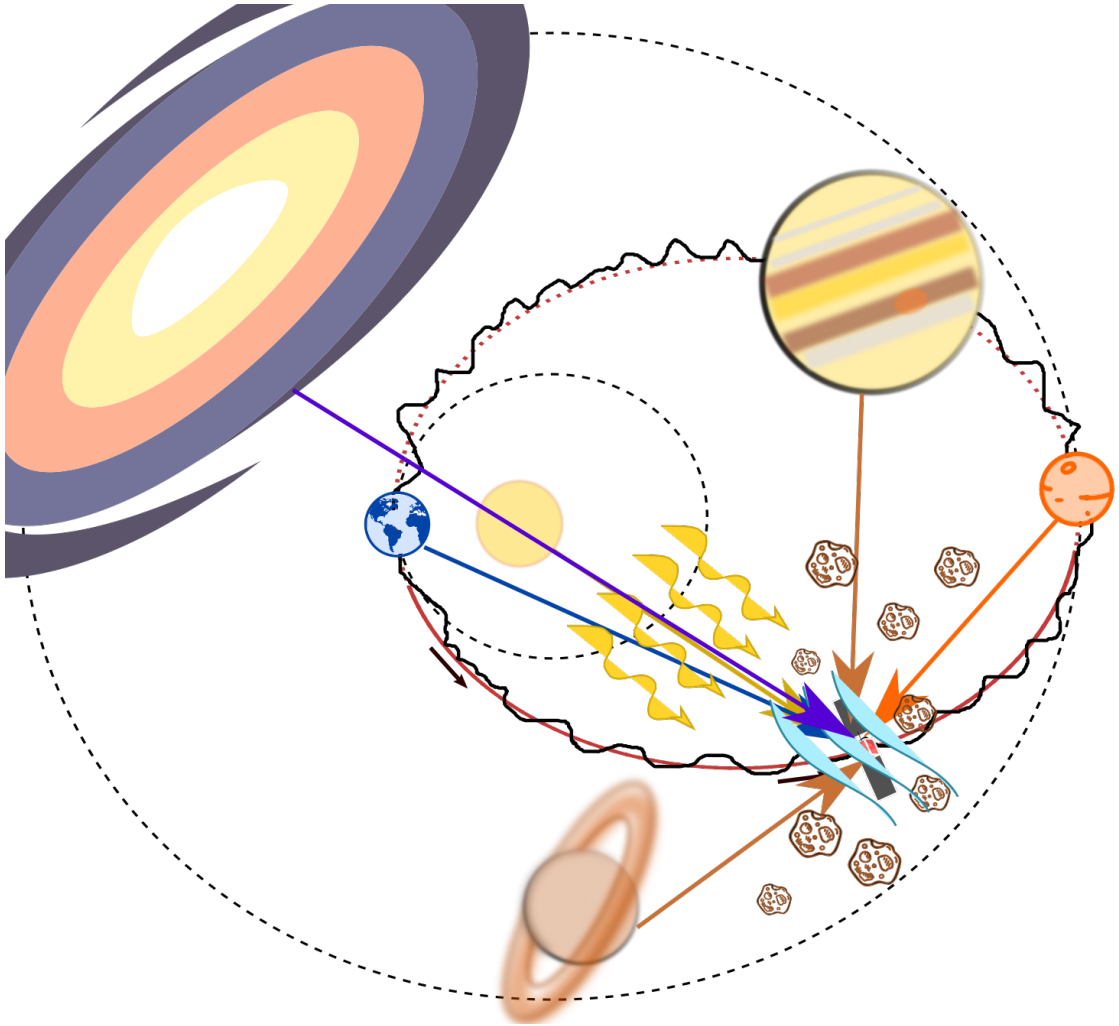

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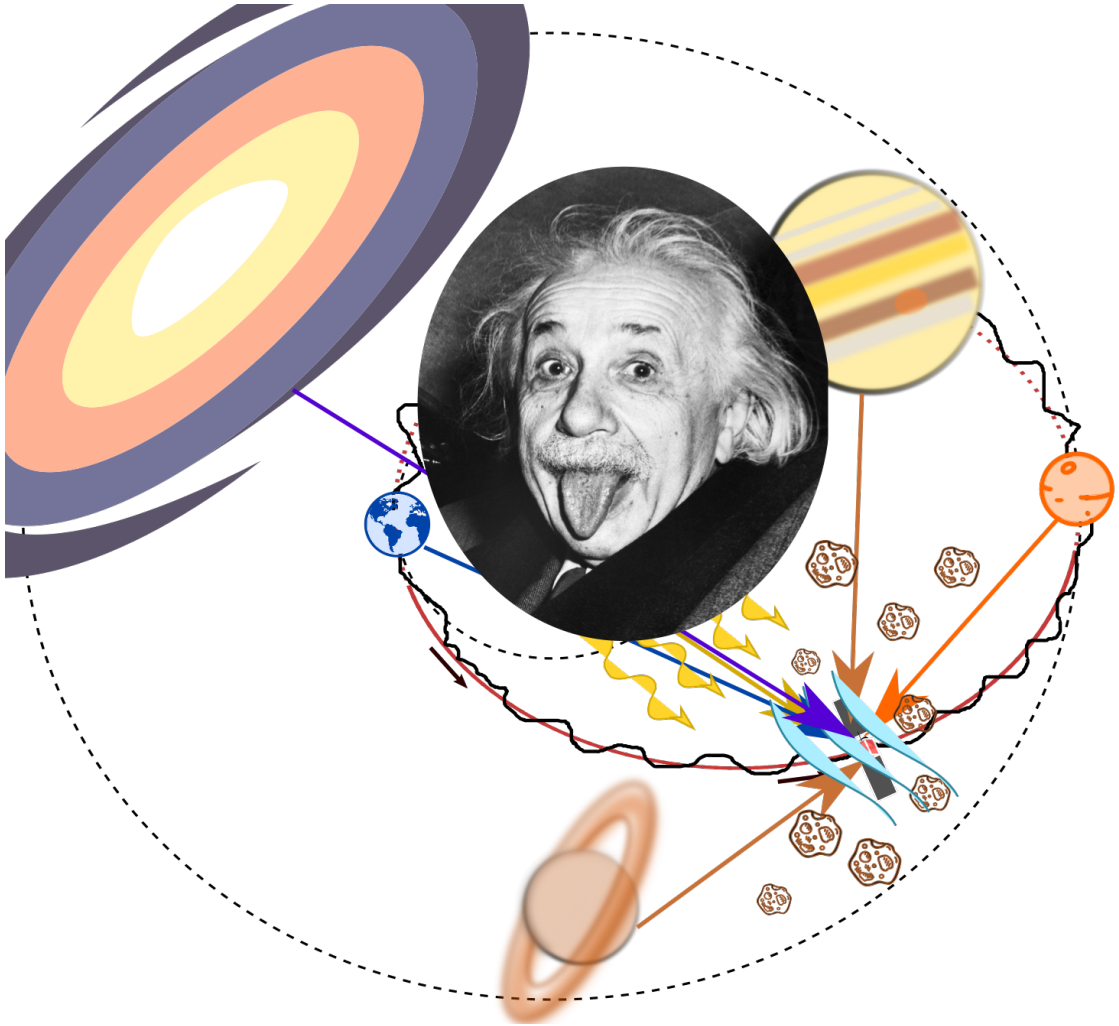

Numerical integrators

We can make approximate calculations using 2-body problems, but reality is an *N*-body problem

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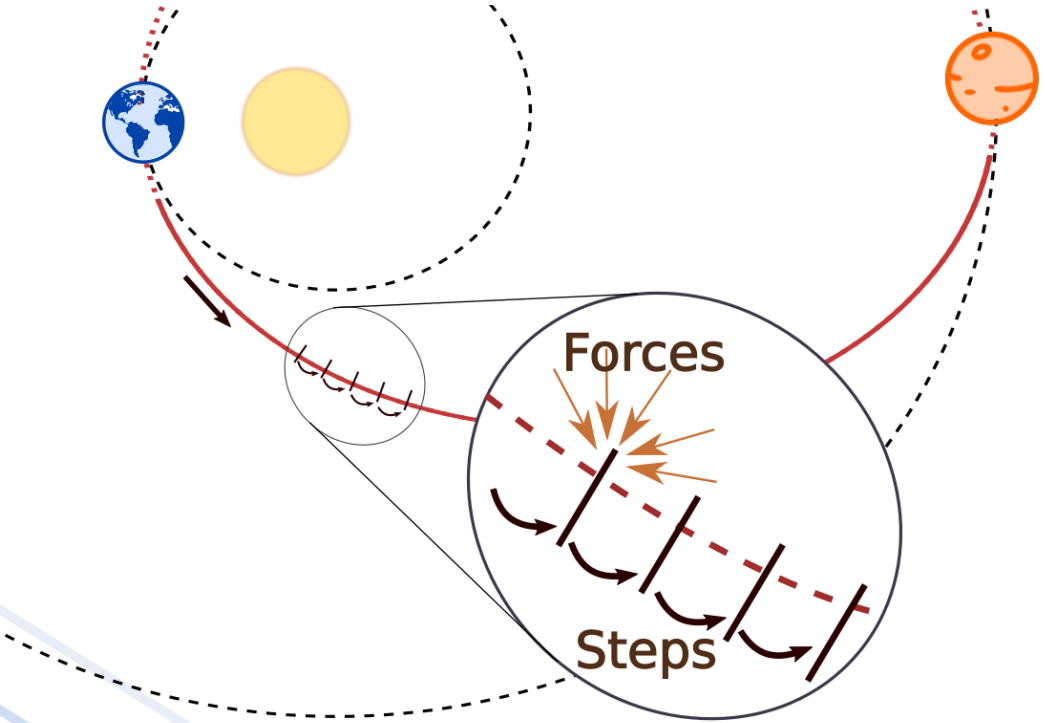

Numerical integrators

$\vec{F} \rightarrow \vec{a} \rightarrow \vec{v}$

Numerical integrators

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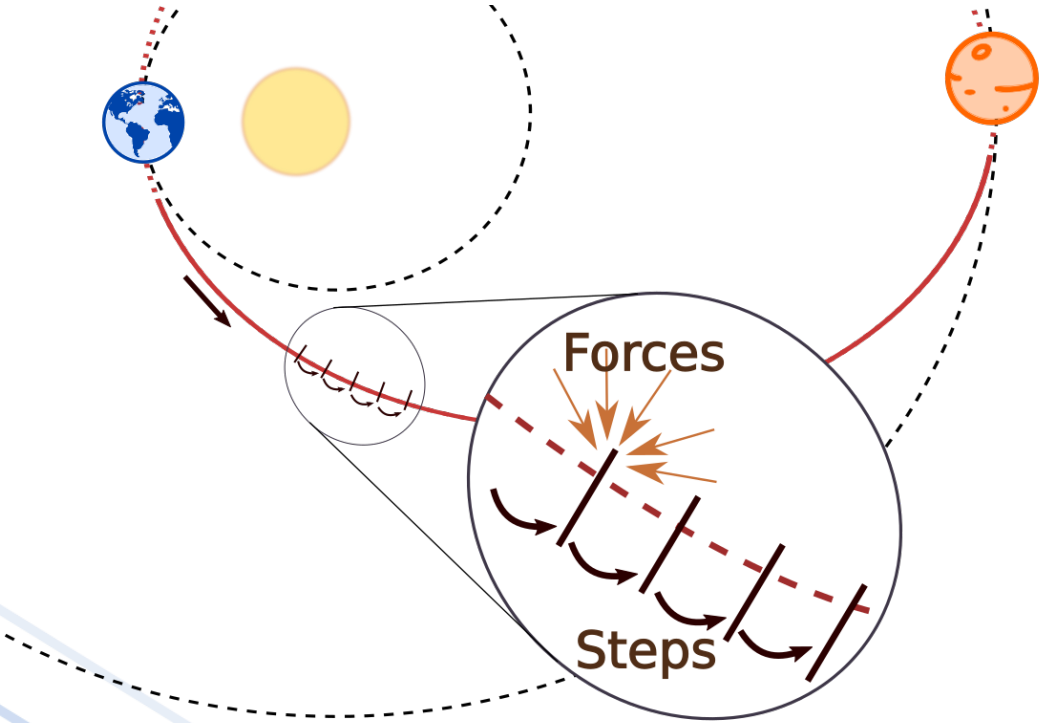


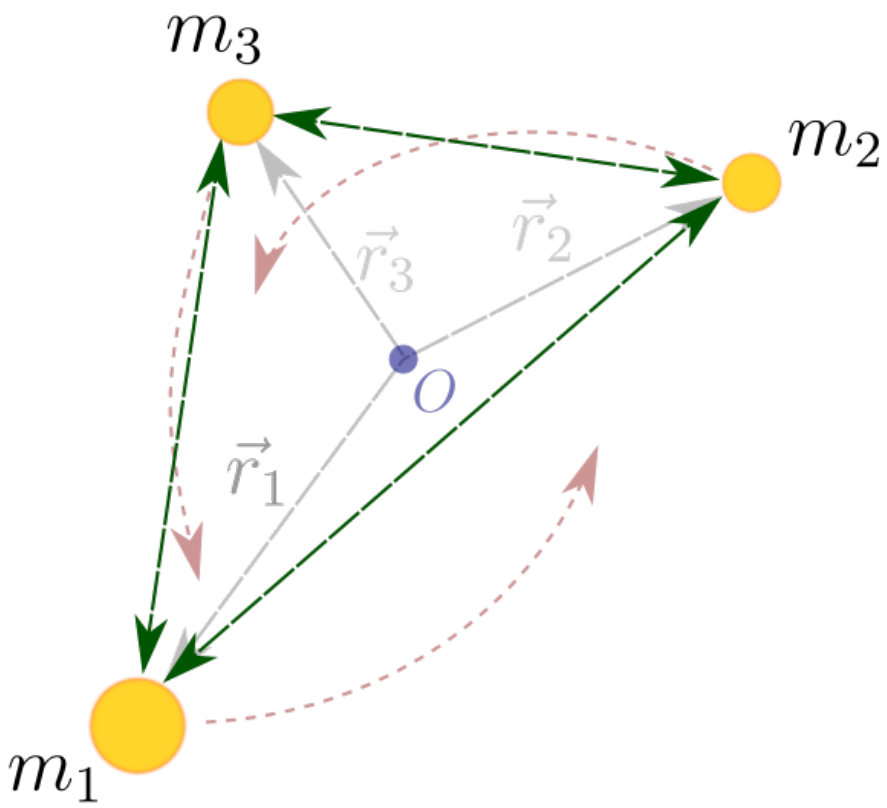
Numerical integrators

$\vec{F} \rightarrow \vec{a} \rightarrow \vec{v}$

1. Propagate in space
2. Calculate effect of all forces in the system
3. Propagate in space
4. Repeat

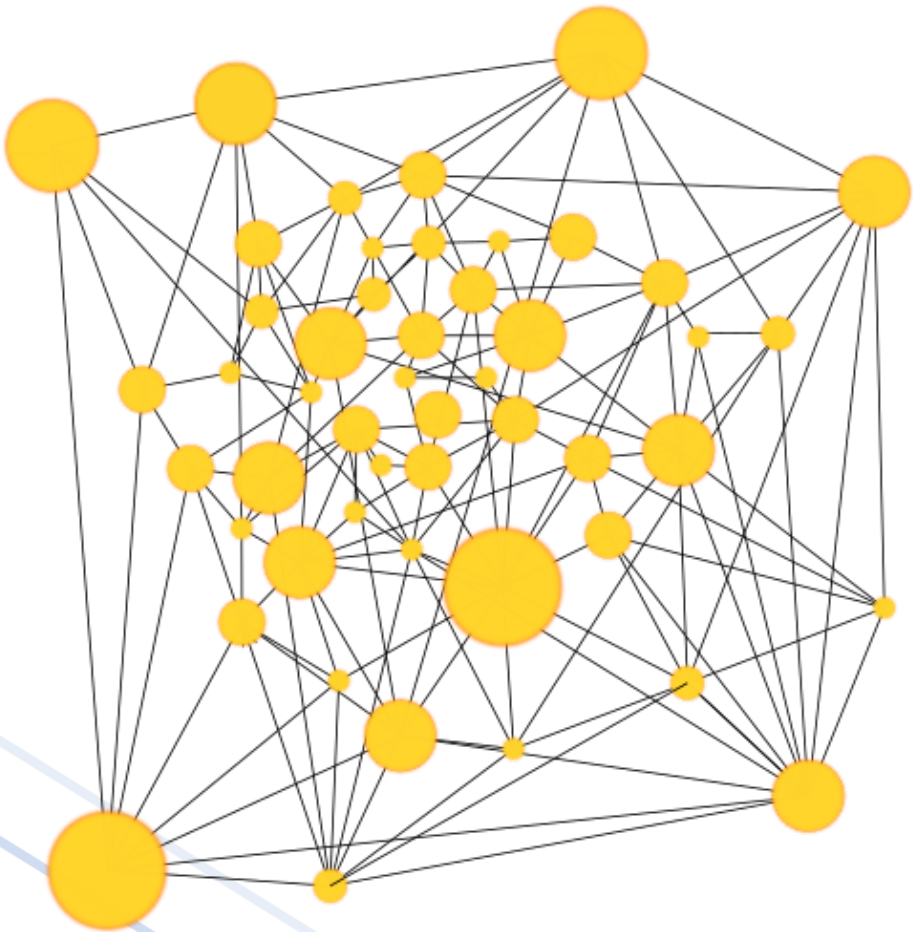
Different types for different cases

Direct integrators



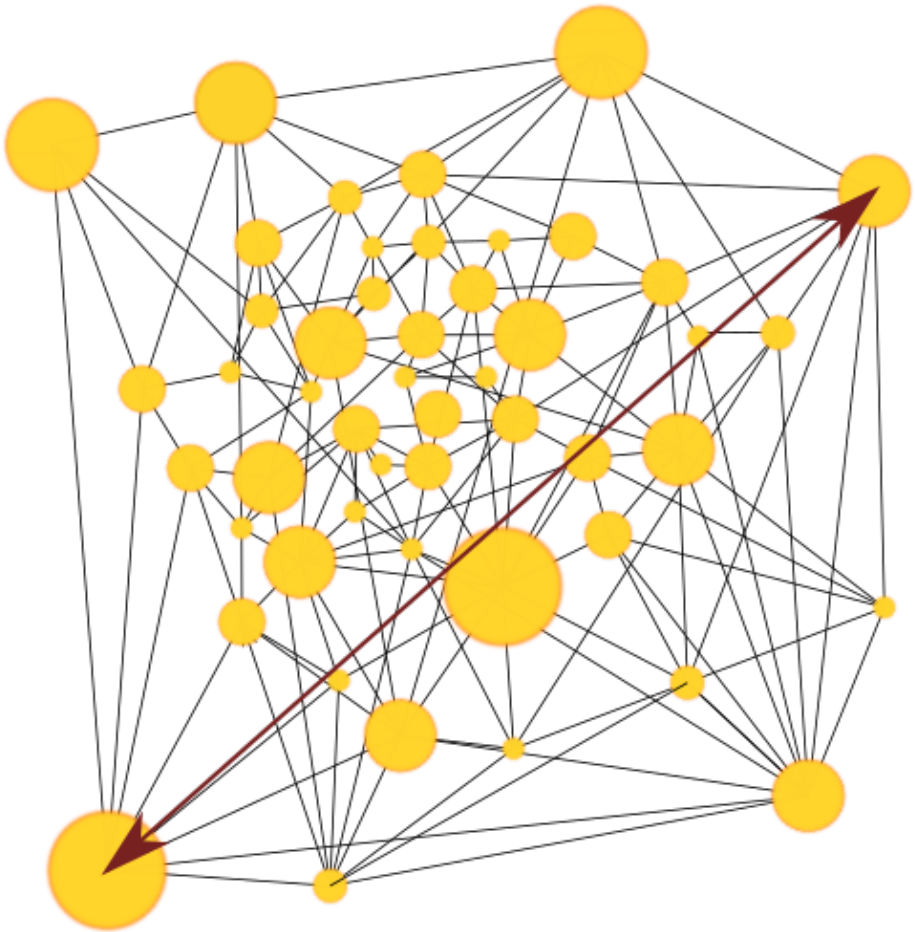
Different types for different cases

Direct integrators



$\longrightarrow$ Very expensive
$\longrightarrow$ Quite accurate

Alternative integrators



Alternative integrators

Tree codes

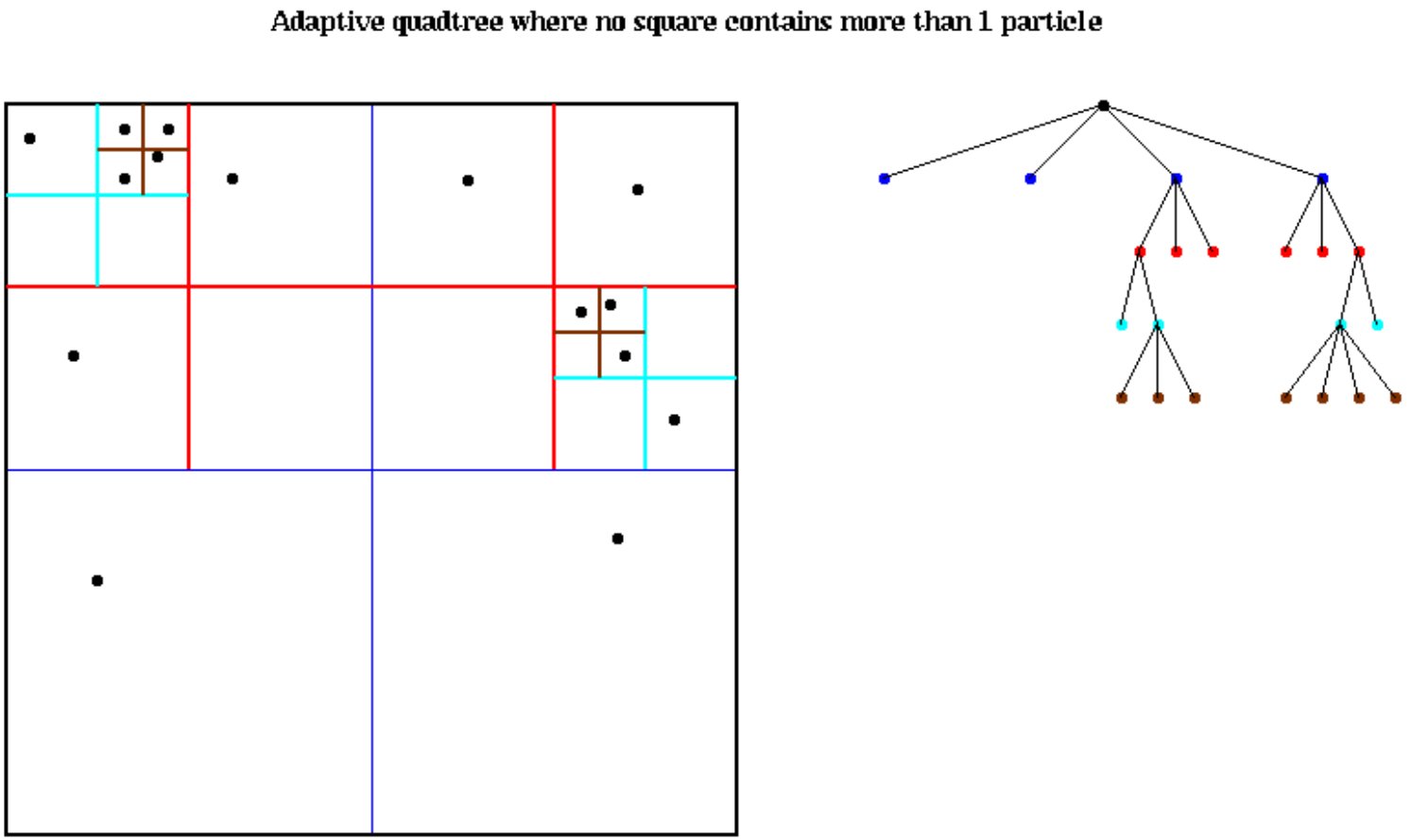


FIGURE – Berkeley

Alternative integrators

Nemesis

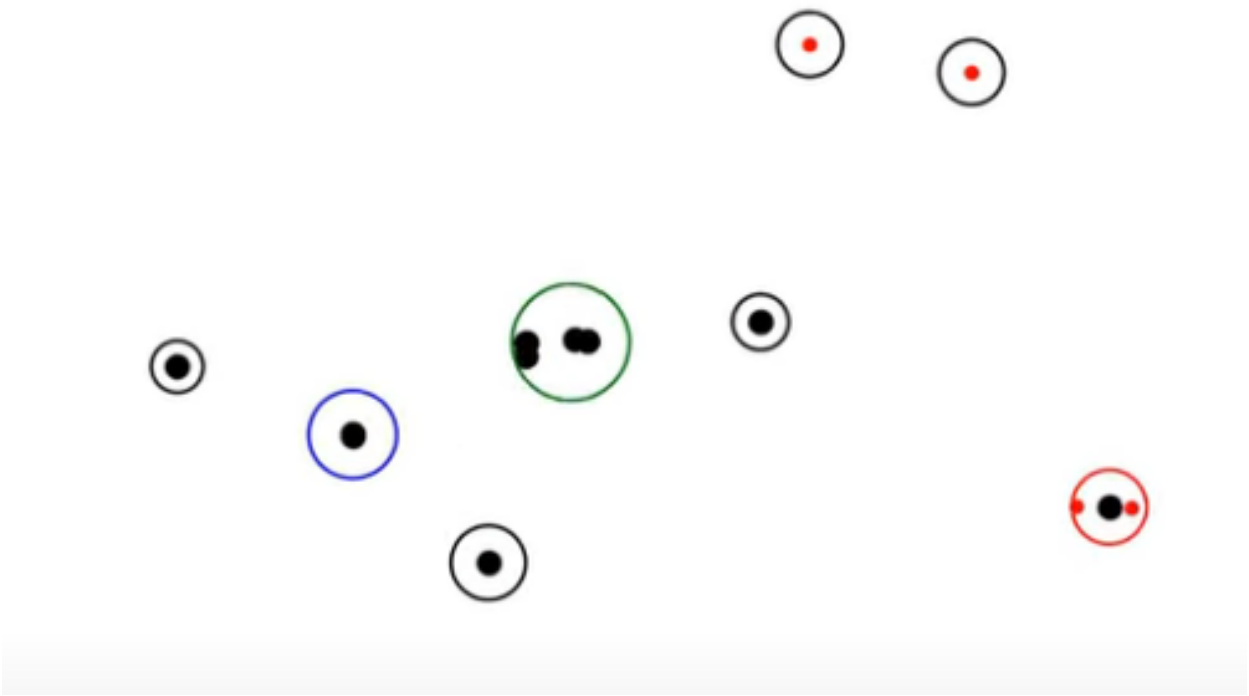


FIGURE – Erwan Hochart

Alternative integrators

Nemesis

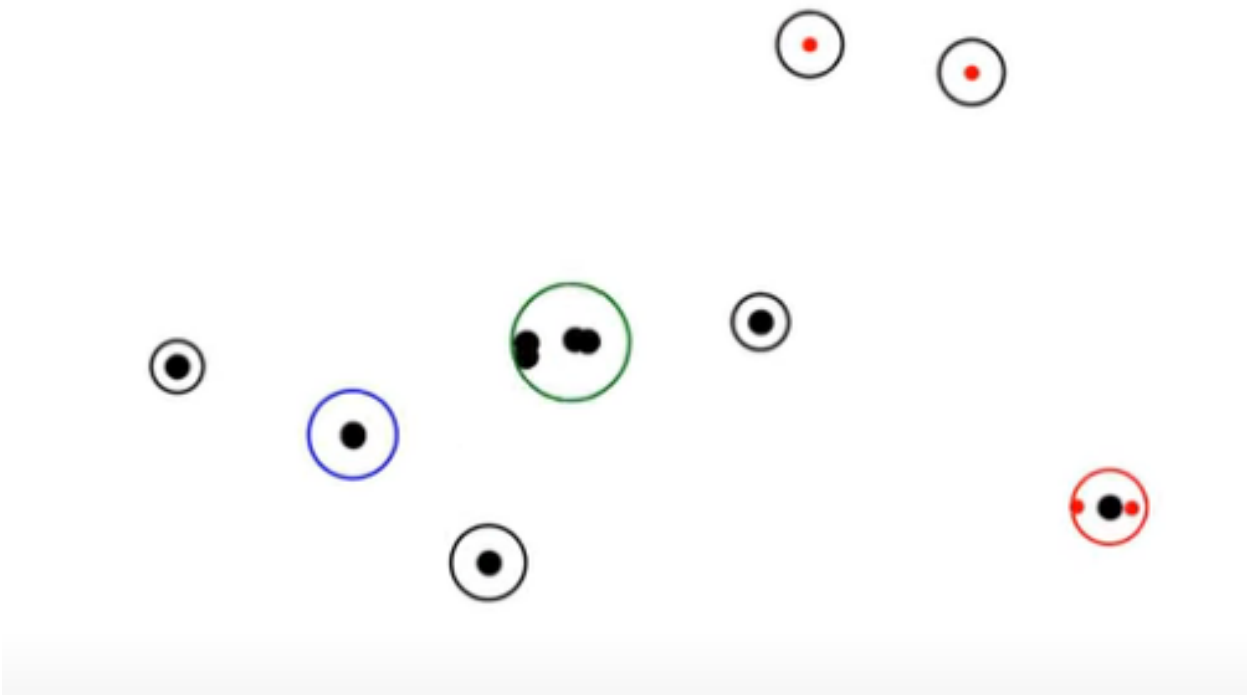


FIGURE – Erwan Hochart

How do we choose small or big time-steps ?

Alternative integrators

Variable-time step integrators

Different time step per **step** ⟵

Different time step per **body** ⟵

FIGURE – NGC 3603 *Source : Banerjee et al.*

Alternative integrators

Tricks

- Ignore particles far away
- Ignore the effect of small particles
- Separate into different codes

Alternative integrators

Tricks

- Ignore particles far away
- Ignore the effect of small particles
- Separate into different codes

Specialized integrators

- Symplectic integrators for planetary systems
- Tree codes for star clusters
- ...

What is Machine Learning ?

"Machine Learning is a method which learns how to perform a specific task and provide accurate results by identifying patterns"

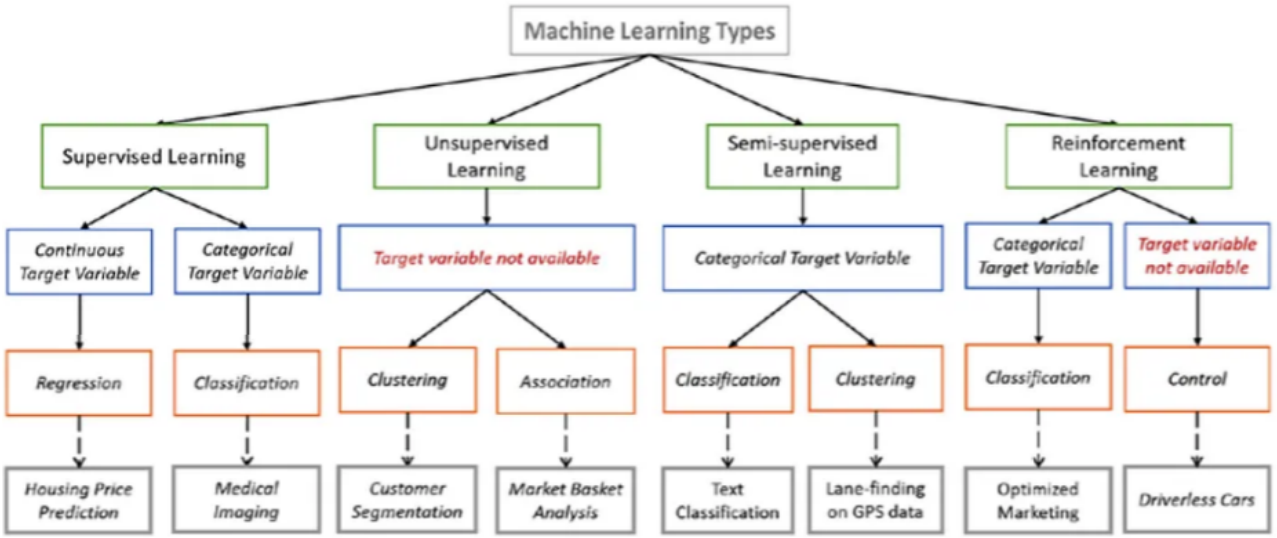


FIGURE – *Source : Priyanka Parashar*

Why do we want to use it ?

If we know the equations

Speed up calculations

Why do we want to use it ?

If we know the equations

Speed up calculations

If we don't know the equations

Find unknown relations

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Optimize for us

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Find unknown relations

If we need to make decisions

Optimize for us

If we don't know the equations

Find new equations

Application 1 : substitution of the integrator

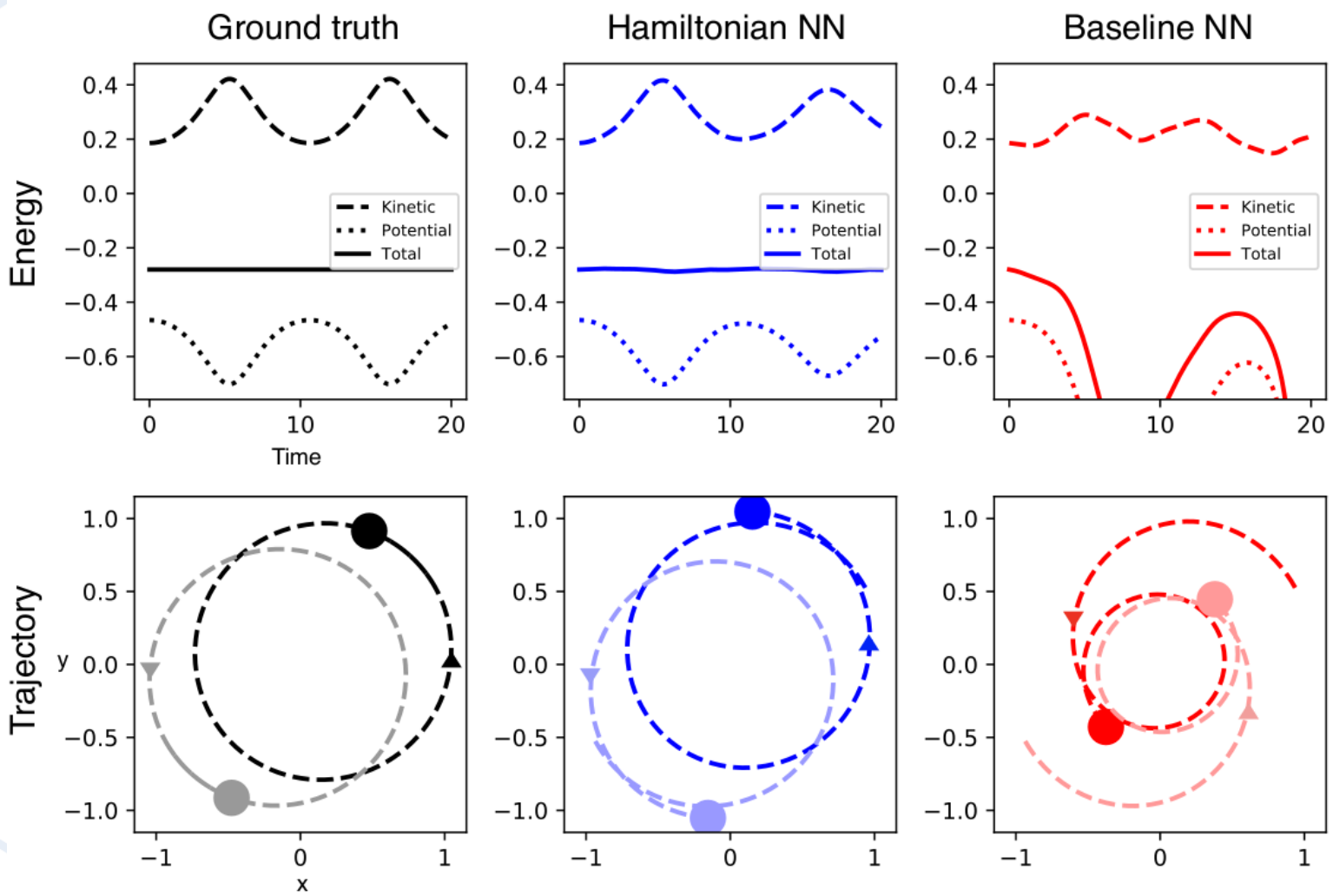

FIGURE – 2-body problem. *Source : Greydanus et al., Hamiltonian Neural Networks, 2019*

Application 1 : substitution of the integrator

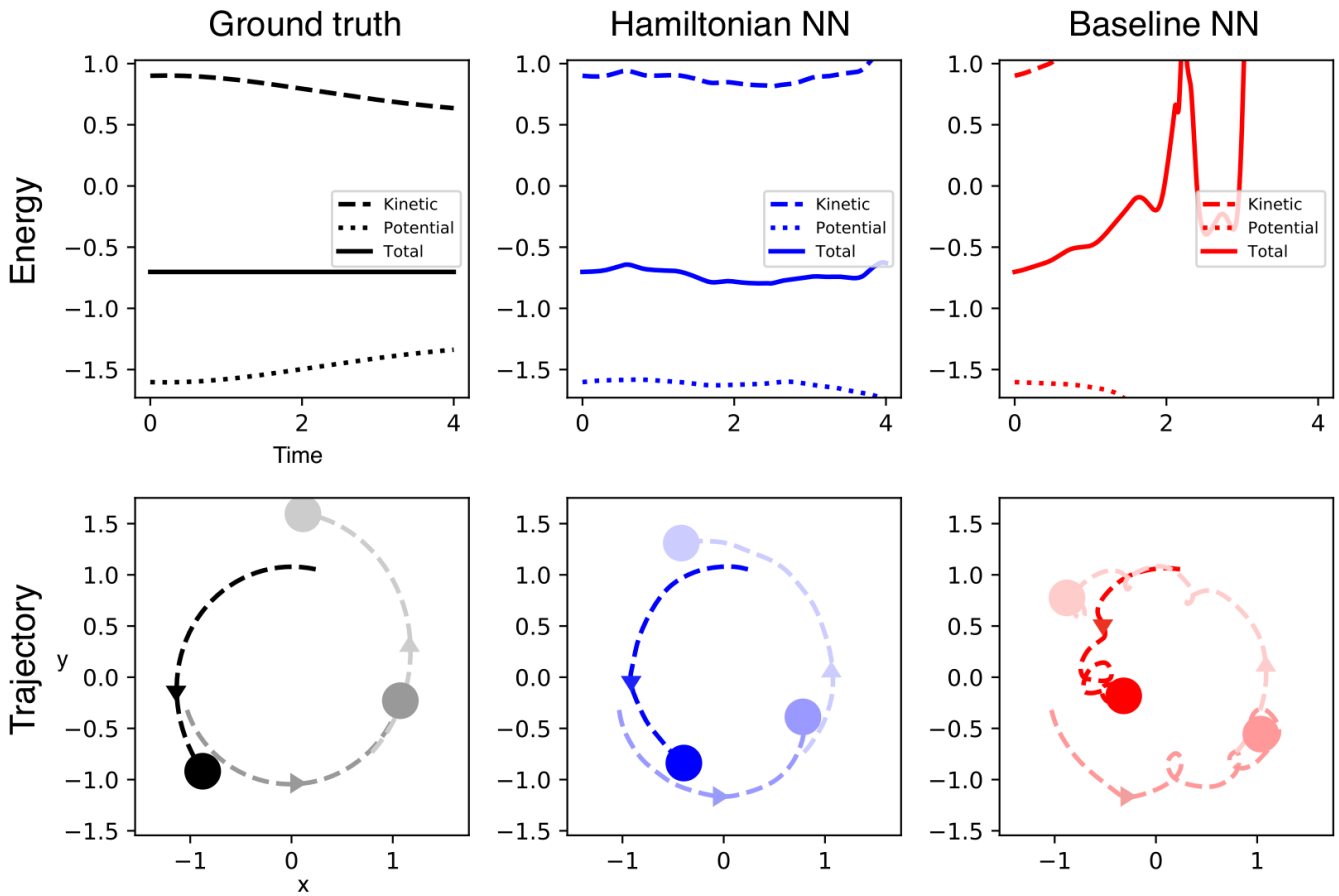

FIGURE – 3-body problem. *Source : Greydanus et al., Hamiltonian Neural Networks, 2019*

Application 2 : infer unknown parameters

```
"InitialConditions":
{
    "units": "si",
    "seed": 1,
    "n_bodies": 5,
    "ranges_mass": [1, 100],
    "radius_cluster": 0.1,
     "virial_ratio": 0.3,
     "disk_radius": [10, 50],
     "mass_disk": 0.02
},
"Integration":
{
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     "subfolder": "",
     "suffix": "",
     "savestate": true,
     "max_steps": 50,
     "check_step": 0.01,
     "t_step_global": 1e-2,
     "t_step_local": 1e-2,
     "integrator_global": "Ph4",
     "integrator_local": "Huayno",
     "max_error_accepted": 1e2,
     "plot": false,
     "bridge": "modified"
```

- ⟶ Initial conditions
- ⟶ Type of integrator
- ⟶ Time step
- ⟶ Integration time

Current challenges

- New development
- Adapting to astrophysics problems
- Training and development time
- Large dynamic range
- Chaotic nature
- Accuracy requirements

Final thoughts

The 2-body problem

- Analytical solution
- Fast!!

The *N*-body problem

- No analytical solution
- Gets more expensive as N increases $\rightarrow$ bigger computers, more time

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Numerical integrators

- Numerical integrators have been developed for >70 years.
- Every time we need more computation power to make larger, more accurate simulations.
- New techniques being involved such as Artificial intelligence.

Questions ?